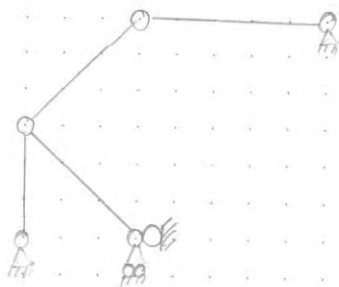


$\Delta_{to}^{CD} = 10^{\circ}C$
 $\alpha t = 1 \times 10^{-5}$
 $W_D = 2 \times 10^2 m$
 $EK = 10^5 kg-m$

MÉTODO DE CROSS

-Imagen Cinemática



Grados de Desplazabilidad = 1

Se procederá a la resolución de las sollicitaciones de momento del sistema estructural mediante la superposición de un problema SIN DESPLAZABILIDAD y un problema CON DESPLAZABILIDAD.

PREPARADOR: REVESCAR J. VIREL R.

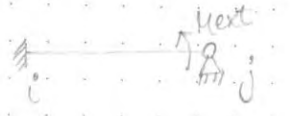
1. Problema sin desplazabilidad.

1.1 Momentos debido a cargas

$$M_{AB}^E = \frac{q \times l^2}{20} = \frac{300 \times (3)^2}{20} = 405 \text{ kgf}\cdot\text{m}$$

$$M_{BA}^E = -\frac{q l^2}{30} = -\frac{300 \times (3)^2}{30} = -270 \text{ kgf}\cdot\text{m}$$

El extremo B es articulado:



$$M_{ij}^j = M_{ij}^E + \frac{1}{2} (M_{ext} - M_{ji}^E)$$

$$M_{AB}^j = M_{AB}^E + \frac{1}{2} (M_{BA}^{ext} - M_{BA}^E) = 405 + \frac{1}{2} (0 - (-270)) = 540 \text{ kgf}\cdot\text{m}$$

$$M_{BE}^E = M_{EB}^E = 0 \quad M_{BE}^j = M_{BE}^E + \frac{1}{2} (M_{EB}^{ext} - M_{EB}^E) = 0 + \frac{1}{2} \times (-2000 - 0) = -1000 \text{ kgf}\cdot\text{m}$$

$$M_{BC}^E = \frac{q \times l^2}{12} = \frac{600 \times (4)^2}{12} = 800; \quad M_{CB}^E = -\frac{q \times l^2}{12} = -800$$

$$M_{CD}^E = \frac{M}{4} = \frac{4000}{4} = 1000; \quad M_{DC}^E = \frac{M}{4} = 1000$$

NOTA: Importante recordar que:

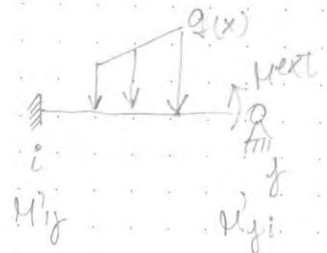
CONDICIÓN EMPOTRADA-EMPOTRADA

$$\left. \begin{aligned} M_{ij} &= M_{ij}^E + 4EK\theta_i + 2EK\theta_j - 6EK\psi_{ij} \\ M_{ji} &= M_{ji}^E + 2EK\theta_i + 4EK\theta_j - 6EK\psi_{ij} \end{aligned} \right\}$$



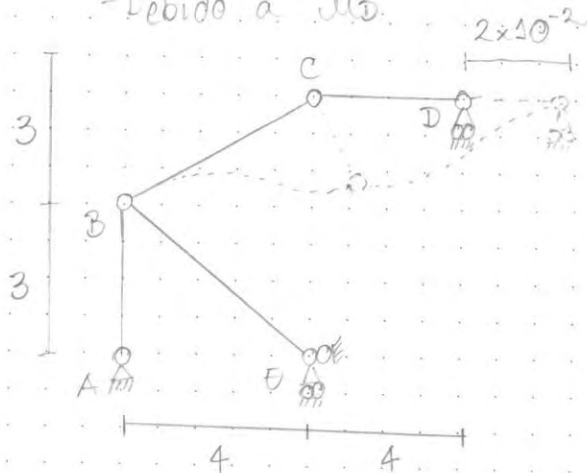
CONDICIÓN EMPOTRADA-ARTICULADA

$$\left. \begin{aligned} M_{ij} &= M_{ij}^E + \frac{1}{2} (M_{ji}^{ext} - M_{ji}^E) + 3EK(\theta_i - \psi_{ij}) \\ M_{ji} &= M_{ji}^{ext} \text{ (VALOR CONOCIDO)} \end{aligned} \right\}$$



1.2 Momentos debido a movimientos de apoyo

- Debido a ΔD .



$$\psi_{bc} = \frac{-2 \times 10^{-2}}{3}$$

$$\delta_c^v = \frac{2 \times 10^{-2} \times 4}{3} = \frac{8}{3} \times 10^{-2}$$

$$\psi_{cd} = \frac{\delta_c^v}{d_{he}^{\text{raro}}} = \frac{\frac{8}{3} \times 10^{-2}}{4} = \frac{2}{3} \times 10^{-2}$$

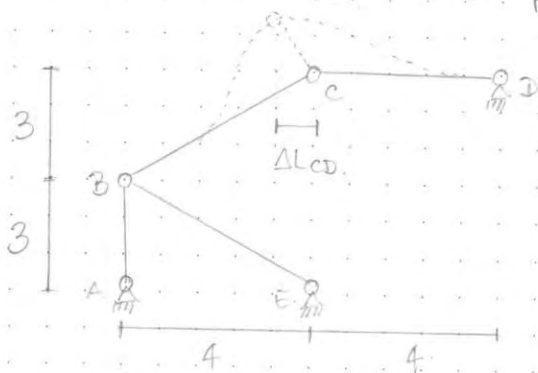
$$M_{bc}^1 = -6EK \psi_{bc} = -6 \times 2 \times 10^5 \times \left(-\frac{2}{3} \times 10^{-2}\right) = 8000 \text{ kgf-m}$$

$$M_{cb}^1 = -6EK \psi_{bc} = 8000 \text{ kgf-m}$$

$$M_{cd}^1 = -6EK \psi_{cd} = -6 \times 2 \times 10^5 \times \left(\frac{2}{3} \times 10^{-2}\right) = -8000 \text{ kgf-m}$$

$$M_{dc}^1 = -6EK \psi_{cd} = -8000 \text{ kgf-m}$$

1.3 Momentos debido al ΔL por el Δt_a^{cd} .



$$\Delta L^{cd} = \alpha \times \Delta t_a^{cd} \times L$$

$$\Delta L^{cd} = 1 \times 10^{-5} \times 10 \times 4 = 4 \times 10^{-4}$$

$$\delta_c^h = 4 \times 10^{-4}$$

$$\psi_{bc} = \frac{\delta_c^h}{d_{he}^{\text{raro}}} = \frac{4 \times 10^{-4}}{3}$$

$$\psi_{cd} = \frac{\delta_c^v}{d_{he}^{\text{raro}}} = \frac{\frac{16 \times 10^{-4}}{3}}{4} = -\frac{4}{3} \times 10^{-4}$$

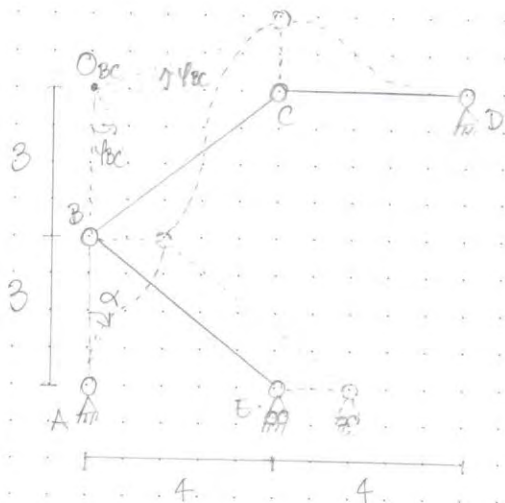
$$\delta_c^v = \frac{4}{3} \times 10^{-4} \times 4 = \frac{16}{3} \times 10^{-4}$$

$$M_{bc}^1 = 6EK \psi_{bc} = -6 \times 2 \times 10^5 \times \frac{4}{3} \times 10^{-4} = -160 \text{ kgf-m}; M_{cb}^1 = -6EK \psi_{bc} = -160 \text{ kgf-m}$$

$$M_{cd}^1 = -6EK \psi_{cd} = -6 \times 2 \times 10^5 \times \left(-\frac{4}{3} \times 10^{-4}\right) = 160 \text{ kgf-m}; M_{dc}^1 = -6EK \psi_{cd} = 160 \text{ kgf-m}$$

PREPARADOR: REVECAR J. VIREL R.

2 - Problema con desplazabilidad



Asumiendo una rotación α (rad) en E.

$$p_{AB} = -\alpha$$

$$d_B^h = \alpha \times 3 = 3\alpha$$

$$p_{BC} = \frac{d_B^h}{d_C^h - p_{AB}} = \frac{3\alpha}{3} = \alpha$$

$$d_C^v = \alpha \times 4 = 4\alpha$$

$$p_{CD} = \frac{d_C^v}{d_C^h} = \frac{4\alpha}{4} = -\alpha$$

$$M_{AB} = 3EK \times (-p_{AB}) = 3 \times 3EK \times (-(-\alpha)) = 9EK\alpha = 9000 \times$$

$$M_{BC} = -6EK (p_{BC}) = -6 \times 2EK \times \alpha = -12EK\alpha = -12000 \times$$

$$M_{CB} = -6EK (p_{BC}) = -12EK\alpha = -12000 \times$$

$$M_{CD} = -6EK (p_{CD}) = -6 \times 2EK \times (-\alpha) = 12EK\alpha = 12000 \times$$

$$M_{EC} = -6EK (p_{CD}) = 12EK\alpha = 12000 \times$$

CAMBIO DE
VARIABLE
 $EK\alpha = 1000 \times$

NOTA = El cambio de variable permite trabajar con números más altos que permitirán incrementar la precisión en los resultados.

MOMENTOS	M_{AB}	M_{BC}	M_{CB}	M_{CD}	M_{EC}	M_{DC}
1.1	540	-1000	800	-800	1000	1000
1.2	0	0	8000	8000	-8000	-8000
1.3	0	0	-160	-160	160	160
PRIMARIOS%	540	-1000	8640	7040	-6840	-6840

- Factores de Distribución

$S_{BE} = 4 \times 2EK = 8EK$ $D_{BE} = \frac{8EK}{12EK} = 0,727$

$S_{CE} = 3EK = \frac{3EK}{11EK}$ $D_{CE} = \frac{3EK}{11EK} = 0,273$

$S_{CB} = 4 \times 2EK = 8EK$ $D_{CB} = 1/2$

$S_{CD} = 4 \times 2EK = \frac{8EK}{10EK}$ $D_{CD} = 1/2$

JUNTA	A	B		C		D
EXT. MIEMBRO	AB	BE	BC	CB	CD	DC
DISTRIBUCIÓN	0	0,727	0,727	0,5	0,5	0
TRANSPORTE	0	0	1/2	1/2	1/2	0
PRIMARIOS %D.	540	-1000	8640	7040	-6840	-6840
		+7640		200		
		-2086	-5554	-2777		
			644	1289	1289	645
		-376	-468	-234		
			59	117	117	59
		-16,11	-42,80	-23,50		
			5,40	10,75	10,75	5,40
		-1,47	-3,93	-1,96		
				0,98	0,98	
TOTALES %D.	540	-3280	3280	5423	-5423	-6131
PRIMARIOS %D.	2000	0	-12000	-12000	12000	12000
		-12000		0		
		3276	8724	4362		
			-1091	-2181	-2181	-1091
		298	793	397		
			-92	-198	-198	-92
		27	72	36		
			-2	-18	-18	-2
		2,45	6,54	3,27		
			-0,82	-1,64	-1,64	-0,82
TOTALES %D.	2000	3603	-3604	-2600	2600	10800

PREPARADOR: REYESCAR J. VIREZ R.

Proyecto TEMA 4 - MÉTODO DE CROSS

Fecha JULIO 2012

Página 6

$$M_{AB} = 540 + 2000x$$

$$M_{BE} = -3200 + 3600x$$

$$M_{BC} = 3200 - 3600x \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} 8703 - 13204x \\ \\ \end{array}$$

$$M_{CB} = 5423 - 2600x$$

$$M_{CD} = -5423 + 2600x$$

$$M_{DC} = -6434 + 10800x \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} -11554 + 20400x \\ \\ \end{array}$$

$$(540 + 2000x)(1-x) + (8703 - 13204x)x - (-11554 + 20400x)x - 4000x - 600 \times 4 \times 2x + 1800 \times 2x + 200 \times \frac{3}{2} \times x = 0$$

$$17667 - 42604x = 0 \quad \Rightarrow \quad x = 0,415$$

• Momentos Definitivos

$$M_{AB} = 4275 \text{ Kg-m}$$

$$M_{BE} = -1785 \text{ Kg-m}$$

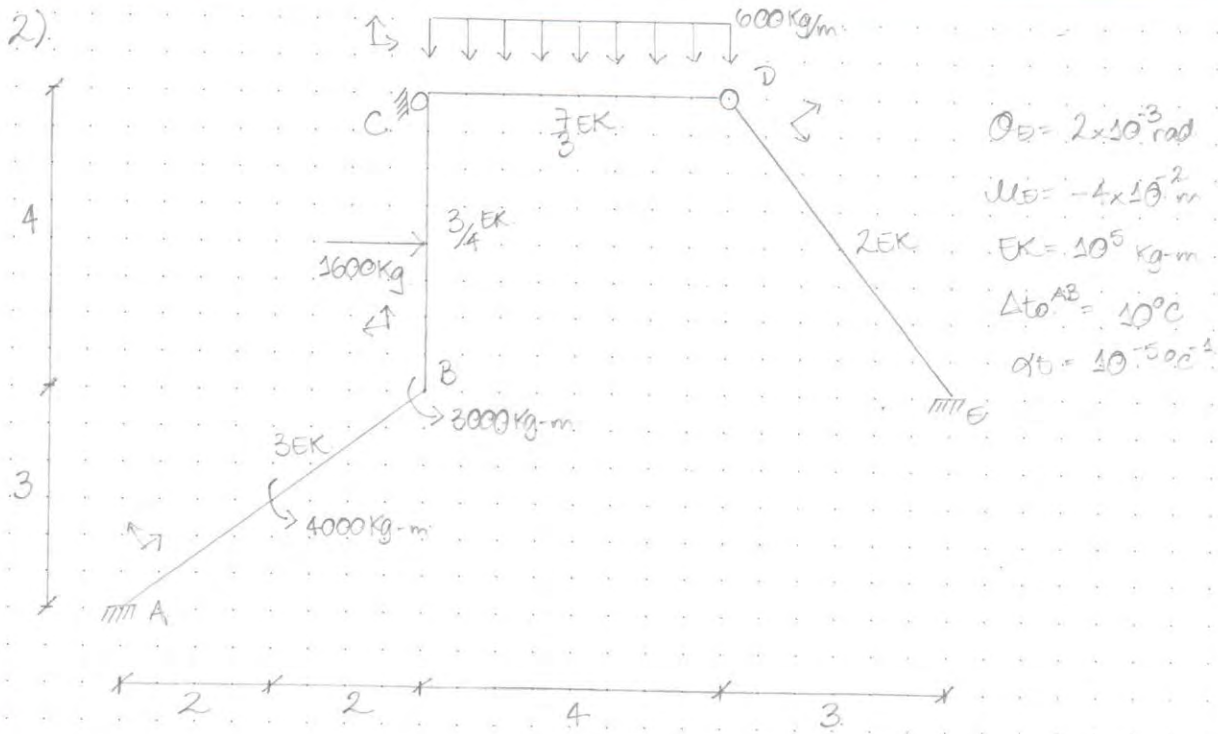
$$M_{BC} = 1785 \text{ Kg-m}$$

$$M_{CB} = 1439 \text{ Kg-m}$$

$$M_{CD} = -1439 \text{ Kg-m}$$

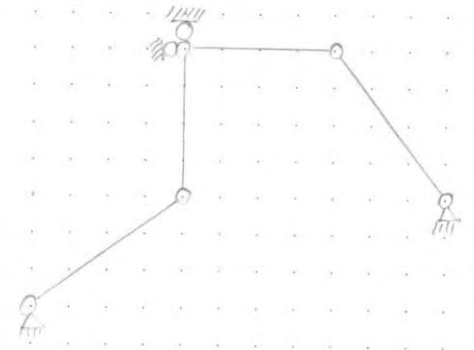
$$M_{DC} = -1649 \text{ Kg-m}$$

PREPARADOR: REBECCA J. VIREL R.



MÉTODO DE CROSS

-Imagen Cinemática



Grados de Desplazabilidad = 1

1- Problema en desplazabilidad

1.1. Momentos debido a cargas

$M_{AB}^E = \frac{4000}{4} = 1000 \text{ kg-m}$; $M_{BA}^E = 1000 \text{ kg-m}$

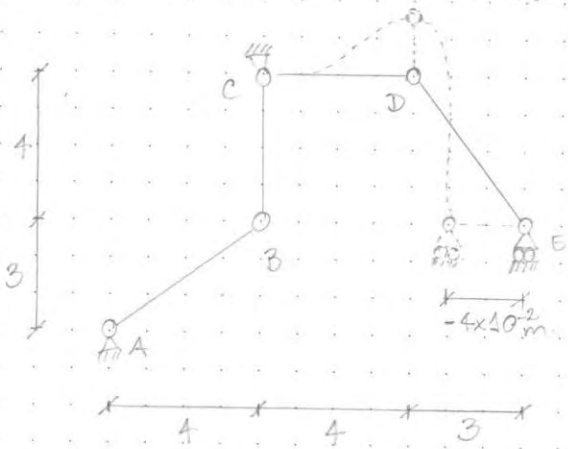
$M_{BC}^E = \frac{P \times L}{8} = \frac{1600 \times 4}{8} = 800 \text{ kg-m}$; $M_{CB}^E = -\frac{P \times L}{8} = -800 \text{ kg-m}$

$M_{CD}^E = \frac{600 \times 4^2}{12} = 800 \text{ kg-m}$; $M_{DC}^E = -800 \text{ kg-m}$

$M_{CD}^* = M_{CD}^E + \frac{1}{2} \times (M_{BC}^E - M_{CB}^E) = 800 + \frac{1}{2} \times (+800) = 1200 \text{ kg-m}$

PREPARADOR RESERVA J VÍDEO 2

1.2 Momentos debido al movimiento de apoyo.



$$\psi_{DE} = \frac{-4 \times 10^{-2}}{4} = -0,01$$

$$\delta_D^V = +0,01 \times 3 = 0,03$$

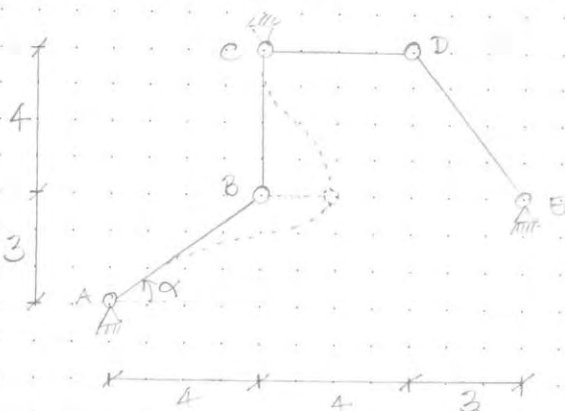
$$\psi_{CD} = \frac{0,03}{4} = 7,5 \times 10^{-3}$$

$$M_{CD} = 3 \times \frac{3}{4} \times 10^5 \times (-7,5 \times 10^{-3}) = -5250 \text{ Kg-m} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Debido a } \psi_{DE}$$

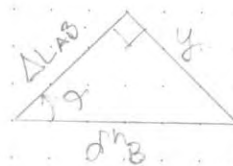
$$M_{ED} = 3 \times 2 \times 10^5 \times (-0,01) = 6000 \text{ Kg-m}$$

$$M_{ED} = 3 \times 2 \times 10^5 \times 2 \times 10^{-3} = 1200 \text{ Kg-m} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Debido a } \delta_D^V$$

1.3 Momentos debido al ΔL por el cambio de temperatura.



$$\Delta L_{AB} = 2 \times 10^{-5} \times 10 \times 5 = 5 \times 10^{-4}$$



$$\cos \alpha = \frac{4}{5} = \frac{\Delta L_{AB}}{d^h_B} \Rightarrow d^h_B = 0,25 \times 10^{-4} \text{ m}$$

$$\psi_{BC} = \frac{0,25 \times 10^{-4}}{4} = 6,25 \times 10^{-5}$$

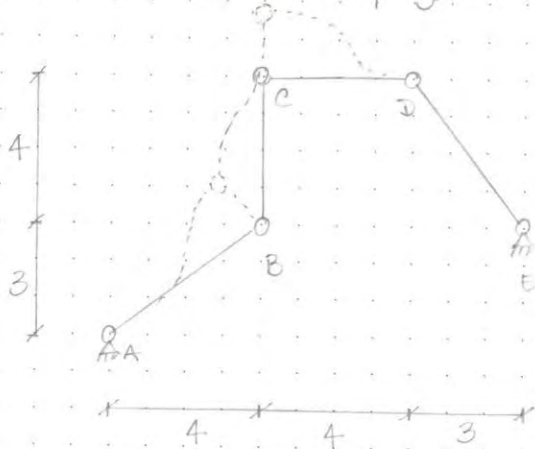
$$\tan \alpha = \frac{3}{4} = \frac{y}{\Delta L_{AB}} \Rightarrow y = 3,75 \times 10^{-4}$$

$$\psi_{BA} = -\frac{3,75 \times 10^{-4}}{5} = -7,5 \times 10^{-5}$$

$$M_{AB} = M_{BA} = -6 \times 3 \times 10^5 \times (-7,5 \times 10^{-5}) = 135$$

$$M_{BC} = M_{CB} = -6 \times \frac{3}{4} \times 10^5 \times 6,25 \times 10^{-5} = -70$$

2. Problema con desplazabilidad



$$\psi_{CD} = -\alpha$$

$$\delta_{V1}^C = \delta_{V1}^B = 4\alpha$$

$$\psi_{AB} = \frac{4\alpha}{4} = \alpha$$

$$\delta_{h1}^B = 3\alpha \Rightarrow \psi_{BC} = -\frac{3}{4}\alpha$$

Cambio de variable $EK\alpha = 1000X$

$$M_{AB} = M_{BA} = -6 \times 3EK\alpha = -18EK\alpha = -18000X$$

$$M_{BC} = M_{CB} = -6 \times \frac{3}{4}EK\left(-\frac{3}{4}\right)\alpha = \frac{27}{8}EK\alpha = 3375X$$

$$M_{CD} = 3 \times \frac{7}{3}EK\alpha = 7EK\alpha = 7000X$$

MOMENTOS	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}
1.1	1000	1000	800	-800	1200	0
1.2	0	0	0	0	-5250	7200
1.3	135	135	-70	-70	0	0
PRIMARIOS %	1135	1135	730	-870	-4050	7200

- Factores de Distribución

$$S_{BA} = 4 \times 3EK = 12EK \quad D_{BA} = 0,80$$

$$S_{CB} = 4 \times \frac{3}{4}EK = 3EK$$

$$D_{CB} = 0,30$$

$$S_{BC} = 4 \times \frac{3}{4}EK = 3EK \quad D_{BC} = 0,20$$

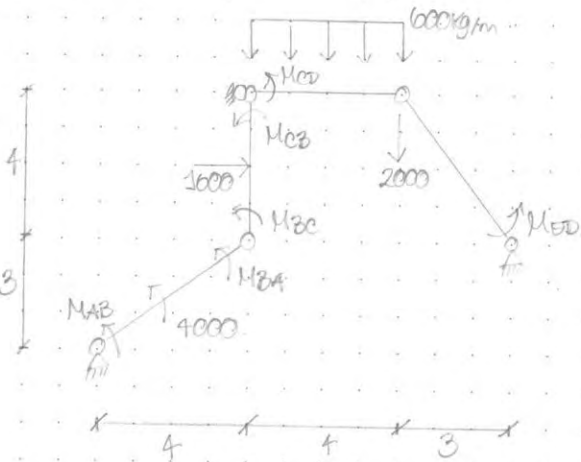
$$S_{CD} = 3 \times \frac{7}{3}EK = 7EK$$

$$D_{CD} = 0,70$$

PREPARADOR: REYESCAR J. VILLALBA

JUNTA	A	B		C		E
EXT. MIEMBRO	AB	BA	BC	CB	CD	ED
DISTRIBUCIÓN	0	1/2	1/2	1/2	0	0
TRANSPORTE	0	0,60	0,20	0,30	0,70	0
PRIMARIOS %	1135	1135	780	-870	-4050	7200
		-1135		-4920		
	158,80	317,16	78	1476	3444	
			79,40	38,70		
			-5,95	-1181	-2779	
	2,38	4,76	1,19	0,59		
TOTALES %	1296	1458	1543	634	-634	7200
PRIMARIOS %	-18000	-18000	3375	3375	7000	0
		-14625		10375		
	5850	11700	2925	1462,5		
			-1775,62	-3551,25	-8206,25	
	710,24	1420,50	355,12	177,56		
			-2663	-532,6	-124,79	
	10,65	21,30	5,32	2,66		
				-0,88	-1,86	
TOTALES %	-11429	-4858	4858	1413	-1413	0

Trabajo Virtual para desplazar "x".



$$\begin{aligned} \varphi_{AB} &= \alpha \\ \varphi_{BC} &= -\frac{3}{4}\alpha \\ \varphi_{CD} &= -\alpha \end{aligned}$$

$$\begin{aligned} M_{AB} &= 1296 - 11429x \\ M_{BA} &= 1458 - 4858x \\ M_{BC} &= 1543 + 4858x \\ M_{CB} &= 634 + 1413x \\ M_{CD} &= -634 - 1413x \\ M_{DC} &= 7200 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} 2754 - 16287x \\ 2177 + 6271x \end{array}$$

PREPARADOS POR: J. V. BELP

$$4000x - 1600 \times \frac{3}{4} x \times 2 - 600 \times 4 \times 2x + (2754 - 16207x)x - (2177 + 6271x) \cdot \frac{3}{4} x - (-634 - 1411x) \cdot x = 0$$

$$-1444,75 - 19579,25x = 0 \Rightarrow x = -7,378 \times 10^{-2}$$

o Momentos Definitivos

$$M_{AB} = 2132$$

$$M_{BA} = 1816$$

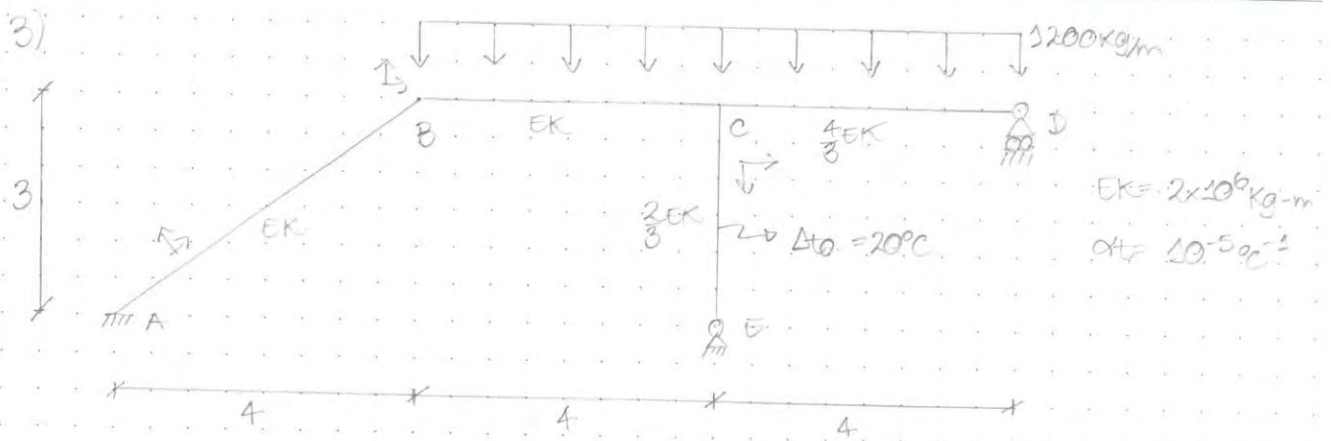
$$M_{BC} = 1185$$

$$M_{CB} = 530$$

$$M_{CD} = -530$$

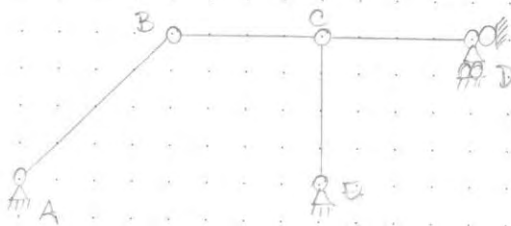
$$M_{DC} = 7200$$

PREPARADOR: REJESAR J. VIREL R.



MÉTODO DE CROSS

- Imagen Cinemática



Grados de Desplazabilidad = 1

1. Problema sin desplazabilidad

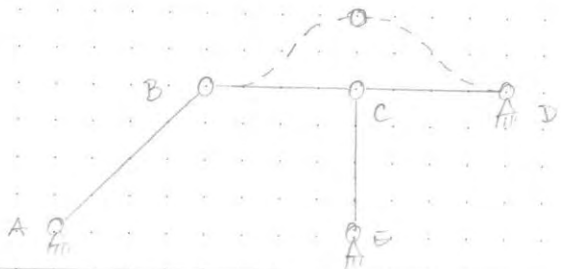
1.1. Momentos debido a cargas

$$M_{BC}^E = \frac{q \cdot l^2}{12} = \frac{1200 \times (4)^2}{12} = 1600 \text{ kg-m}; M_{CB}^E = -\frac{q \cdot l^2}{12} = -1600 \text{ kg-m}$$

$$M_{CD}^E = \frac{q \cdot l^2}{12} = 1600 \text{ kg-m}; M_{DC}^E = -1600 \text{ kg-m}$$

$$M_{CB}^A = 1600 + \frac{1}{2} (0 - (-1600)) = 2400 \text{ kg-m}$$

1.2. Momentos debido al Δt del Δt



$$\Delta L_{CE} = 10^{-5} \times 20 \times 3 = 6 \times 10^{-4}$$

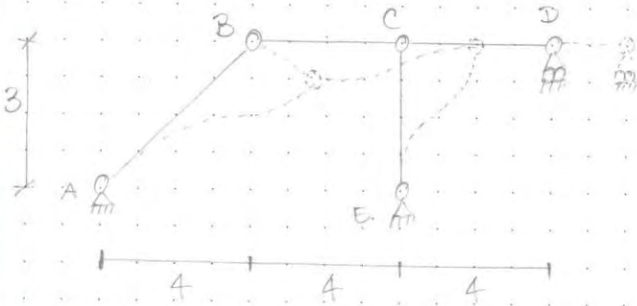
$$P_{BC} = -P_{CD} = \frac{6 \times 10^{-4}}{4} = 1,5 \times 10^{-4}$$

PREPARADOR: REVISOR: J. V. RUIZ

$$M_{BC} = M_{CB} = -6 \times 2 \times 10^6 \times 1,5 \times 10^{-4} = -1800$$

$$M_{CD} = 3 \times 2 \times \frac{4}{3} \times 10^6 \times 1,5 \times 10^{-4} = 1200$$

2.- Problema con desplazabilidad



$$\psi_{EC} = -\alpha$$

$$\delta_{DC}^h = \delta_{CB}^h = 3\alpha \Rightarrow \psi_{AB} = -\frac{3\alpha}{3} = -\alpha$$

$$\delta_{CB}^v = 4\alpha \Rightarrow \psi_{BC} = \frac{4\alpha}{4} = \alpha$$

Cambio de Variable: $EK\alpha = 1000X$

$$M_{AB} = M_{BA} = -6 \times EK \times L \times \alpha = 6EK\alpha = 6000X$$

$$M_{BC} = M_{CB} = -6 \times EK \times \alpha = -6EK\alpha = -6000X$$

$$M_{CD} = 3 \times \frac{2}{3} EK \alpha = 2EK\alpha = 2000X$$

- Factores de Distribución

$$S_{BA} = 4EK$$

$$D_{BA} = 0,50$$

$$S_{CB} = 4EK$$

$$D_{CB} = 0,40$$

$$S_{BE} = \frac{4EK}{0EK}$$

$$D_{BE} = 0,50$$

$$S_{CD} = 3 \times \frac{2EK}{3}$$

$$D_{CD} = 0,20$$

$$S_{ED} = \frac{3 \times 4EK}{3}$$

$$D_{ED} = 0,40$$

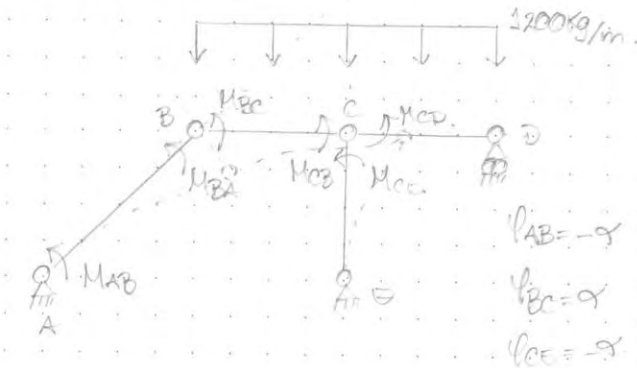
$$10EK$$

MOMENTOS	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{ED}
1.1	0	0	1600	-1600	0	2400
1.2	0	0	-1800	-1800	0	1200
PRIMARIOS %	0	0	-200	-2400	0	3600

JUNTA	A	B		C		
EXT. MIEMBRO	AB	BA	BC	CB	CE	CD
DISTRIBUCIÓN	0	0,50	0,50	0,40	0,20	0,40
TRANSPORTE	0	1/2	1/2	1/2	0	0
PRIMARIOS %	0	0	-200	-3400	0	3600
		-200		200		
	50	100	100	50		
			-50	-100	-50	-100
	12,5	25	25	12,50		
			-2,50	-5	-2,50	-5
		1,25	1,25			
TOTALES %	63	126	-126	-3443	-53	3495
PRIMARIOS %	6000	6000	-6000	-6000	2000	0
		0		-4000		
			800	1600	800	1600
	-200	-400	-400	-200		
			40	80	40	80
	-10	-20	-20	-10		
			2	4	2	4
	-0,50	-1	-1	-0,50		
TOTALES %	5790	5579	-5579	-4526	2842	1684

- Trabajo Virtual para despejar "x"

$$\begin{aligned} M_{AB} &= 63 + 5790x \\ M_{BA} &= 126 + 5579x \\ M_{BC} &= -126 - 5579x \\ M_{CB} &= -3443 - 4526x \\ M_{CE} &= -53 + 2842x \\ M_{CD} &= 3495 + 1684x \end{aligned} \quad \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \begin{array}{l} 109 + 11369x \\ -3569 - 10105x \end{array}$$

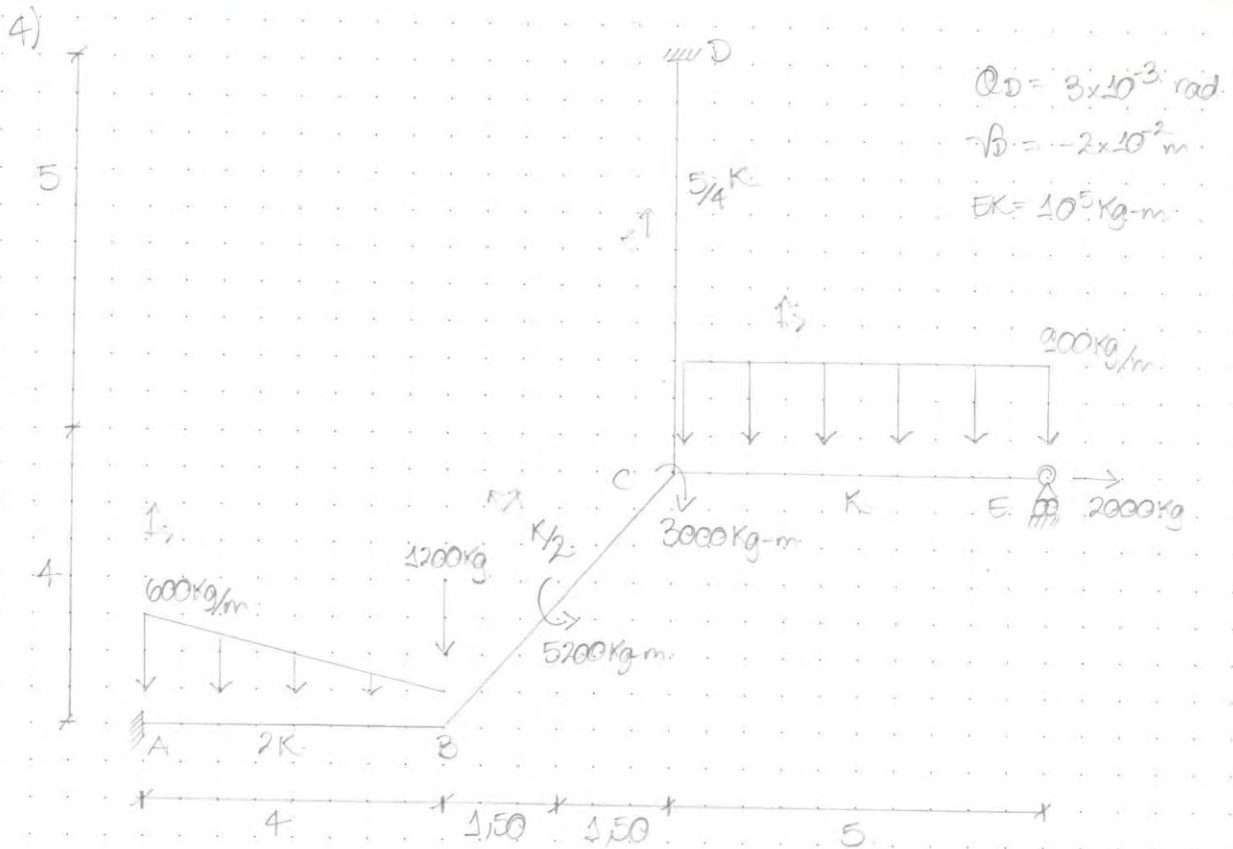


$$0 = 1200 \times 4 \times 2\alpha + (109 + 11369x) \cdot (-\alpha) + (-3569 - 10105x) \cdot (\alpha) + (-53 + 2842x) \cdot (-\alpha)$$

$$5895 - 24316x = 0 \Rightarrow x = 0,242$$

Momentos Definitivos:

$$\begin{aligned} M_{AB} &= 1464 & M_{BC} &= -1476 & M_{CE} &= 635 \\ M_{BA} &= 1476 & M_{CB} &= -4538 & M_{CD} &= 3903 \end{aligned}$$



Método de Cross

1) Problema sin desplazabilidad

1.1 - Momentos debido a cargas

$$M_{AB}^E = \frac{qL^2}{20} = \frac{600 \times (4)^2}{20} = 480 \text{ kg-m}; \quad M_{BA}^E = -\frac{qL^2}{30} = -\frac{600 \times (4)^2}{30} = -320 \text{ kg-m}$$

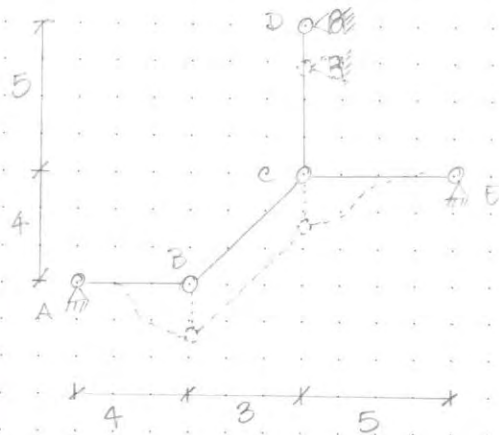
$$M_{BC}^E = \frac{H}{4} = \frac{5200}{4} = 1300 \text{ kg-m}; \quad M_{CB}^E = -1300 \text{ kg-m}$$

$$M_{CE}^E = \frac{qL^2}{12} = \frac{200 \times (5)^2}{12} = 416.67 \text{ kg-m}; \quad M_{EC}^E = -416.67 \text{ kg-m}$$

$$M_{CE}^* = M_{CE}^E + \frac{1}{2} (M_{CB}^E + M_{EC}^E) \Rightarrow M_{CE}^* = 416.67 + \frac{1}{2} \times (-1300 + 416.67) = 283.33 \text{ kg-m}$$

PREPARADOR: PEDRO ANGEL...

1.2 Momentos debido a movimientos de apoyo



$$\varphi_{AB} = \frac{2 \times 10^{-2}}{4} = -5 \times 10^{-3}$$

$$\varphi_{CE} = \frac{2 \times 10^{-2}}{5} = 4 \times 10^{-3}$$

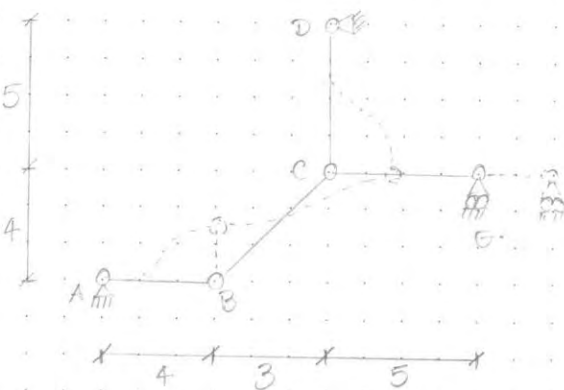
$$M_{AB} = M_{BA} = -6 \times 2 \times 10^5 \times (-5 \times 10^{-3}) = 6000 \text{ kg-m}$$

$$M_{BC} = M_{CB} = 3 \times 10^5 \times (-4 \times 10^{-3}) = -1200 \text{ kg-m}$$

$$M_{CD} = 2 \times \frac{5}{4} \times 10^5 \times 3 \times 10^{-3} = 750 \text{ kg-m}$$

$$M_{DE} = 4 \times \frac{5}{4} \times 10^5 \times 3 \times 10^{-3} = 1500 \text{ kg-m}$$

2. Problema con Desplazabilidad



$$\varphi_{AB} = \alpha$$

$$\delta_B^v = 4\alpha \Rightarrow \varphi_{BC} = -\frac{4}{3}\alpha$$

$$\delta_C^v = \frac{4}{3}\alpha \times 4 = \frac{16}{3}\alpha \Rightarrow \varphi_{CD} = \frac{\frac{16}{3}\alpha}{5} = \frac{16}{15}\alpha$$

Cambio de Variable $EK\alpha = 1000x$

$$M_{AB} = M_{BA} = -6 \times 2 \times EK\alpha = -12EK\alpha = -12000x$$

$$M_{BC} = M_{CB} = -6 \times EK \times \left(-\frac{4}{3}\alpha\right) = 4EK\alpha = 4000x$$

$$M_{CD} = M_{DC} = -6 \times 5EK \times \frac{16}{15}\alpha = -8EK\alpha = -8000x$$

PREPARADOR PEDROSAZ I. / 12/11/22

Proyecto TERA A - MÉTODO DE CROSS

Fecha OCTUBRE 2012

Página 17

Factores de Distribución

$$\begin{aligned}
 S_{BA} &= 4 \times 2EK = 8EK & D_{BA} &= 0,80 & S_{CB} &= 2EK & D_{CB} &= 0,20 \\
 S_{BC} &= 4 \times EK = \frac{2EK}{2} = 10EK & D_{BC} &= 0,20 & S_{CD} &= 4 \times \frac{5EK}{4} = 5EK & D_{CD} &= 0,50 \\
 & & & & S_{CE} &= 3EK & D_{CE} &= 0,30 \\
 & & & & & & & 10EK
 \end{aligned}$$

MOMENTOS	M _{AB}	M _{BA}	M _{BC}	M _{CB}	M _{CE}	M _{CD}	M _{CE}
4.1	480	-320	1300	1300	2813	0	0
4.2	6000	6000	0	0	-1200	750	1500
PRIMARIOS %	6480	5680	1300	1300	1613	750	1500

JUNTA	A		B		C		D
EXT. MIEMBRO	AB	BA	BC	CB	CE	CD	DE
DISTRIBUCIÓN	0	0,80	0,20	0,20	0,30	0,50	0
TRANSPORTE	0	1/2	1/2	1/2	0	1/2	0
PRIMARIOS %	6480	5680	1300	1300	1613	750	1500
		6900			8663		
	-2792	-5584	-1396	-698			
			-597	-1193	-1790	-2982	-1491
	232	478	119	60			
			-6	-12	-18	-30	-15
	2,40	4,80	1,20	0,60			
TOTALES %	3929	579	-579	-542	-125	-2262	-6
PRIMARIOS %	-12000	-12000	4000	4000	0	-8000	-8000
		-8000			-4000		
	3200	6400	1600	800			
			320	+640	260	1600	800
	-128	-256	-64	-32			
			312	6,4	2,60	16	8
	-1,20	-2,60	-0,60				
TOTALES %	-8929	-5859	5859	5414	970	-6384	-2192

PREPARADOR: REYESSE J. VIVER D.

- Principio de Trabajo Virtual para desplazar "x"

$$M_{AB} = 3222 - 8922x \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} 4508 \\ -14780x \end{array}$$

$$M_{BA} = 579 - 5852x$$

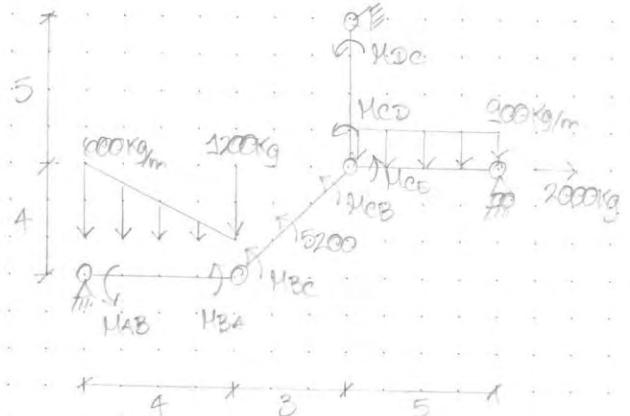
$$M_{BC} = -579 + 5852x \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} -1121 \\ +11273x \end{array}$$

$$M_{CB} = -542 + 5414x$$

$$M_{CE} = -195 + 970x$$

$$M_{CD} = -2262 - 6384x \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} -2268 \\ -13576x \end{array}$$

$$M_{DC} = -6 - 7192x$$



$$V_{AB} = x; \quad V_{BC} = -\frac{4}{3}x; \quad V_{CD} = \frac{16}{15}x; \quad V_{CE} = 0$$

$$(4508 - 14780x)x - \frac{600 \times 4 \times x \times \frac{1}{3} \times 4}{2} - 1200 \times 4x - (-1121 + 11273x) \times \frac{4}{3}x - 5200 \times \frac{4}{3}x$$

$$+ 2000 \times \frac{16}{3}x + (-2268 - 13576x) \times \frac{16}{15}x = 0$$

$$216180 - 44300x \Rightarrow x = 0,0206$$

o Momentos Definitivos

$$M_{AB} = 3745 \text{ Kg-m}$$

$$M_{BA} = 458 \text{ Kg-m}$$

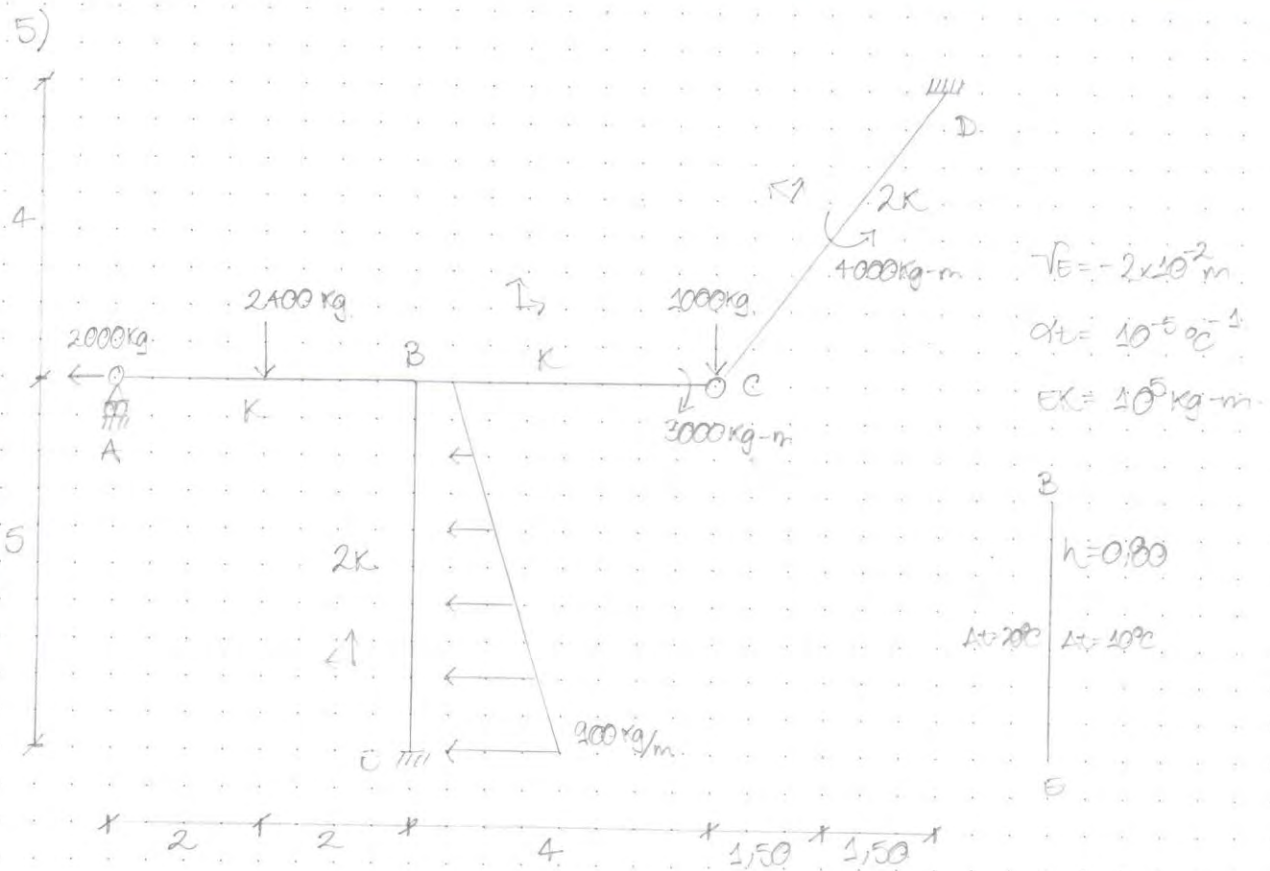
$$M_{BC} = -458 \text{ Kg-m}$$

$$M_{CB} = -430 \text{ Kg-m}$$

$$M_{CE} = -175 \text{ Kg-m}$$

$$M_{CD} = -2394 \text{ Kg-m}$$

$$M_{DC} = -154 \text{ Kg-m}$$



MÉTODO DE CROSS:

1- Problema sin desplazabilidad

1.1. Momentos debido a cargas

$$M_{CB}^E = -\frac{qL^2}{20} = -\frac{900 \times (5)^2}{20} = -1125 \text{ kg-m} \quad M_{BC}^E = \frac{qL^2}{20} = \frac{900 \times (5)^2}{20} = 1125 \text{ kg-m}$$

$$M_{AB}^E = \frac{P \times L}{8} = \frac{2400 \times 4}{8} = 1200 \text{ kg-m} \quad M_{BA}^E = -\frac{P \times L}{8} = -1200 \text{ kg-m}$$

$$M_{BA}^X = -1200 + \frac{1}{2}(-1200) = -1800 \text{ kg-m}$$

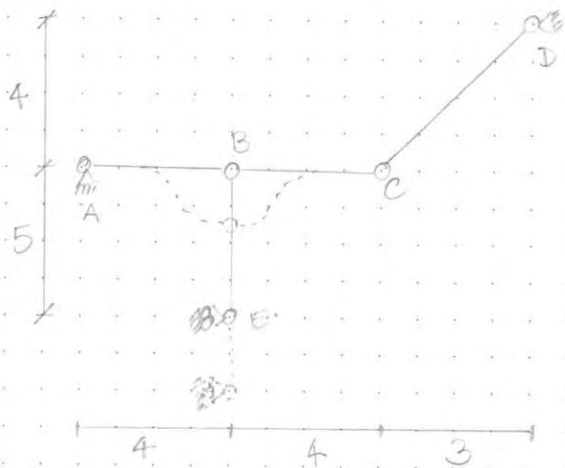
$$M_{CB}^X = \frac{1}{2} \times (-1125) = -562.5 \text{ kg-m}$$

$$M_{CD}^E = 1000 \text{ kg-m} \quad M_{DC}^E = 4000 \text{ kg-m}$$

$$M_{DC}^X = 1000 + \frac{1}{2}(-4000) = 500 \text{ kg-m}$$

PREPARADOR: RODRIGUEZ J. / REC. P.

1.2 Momentos debido al movimiento de apoyo.



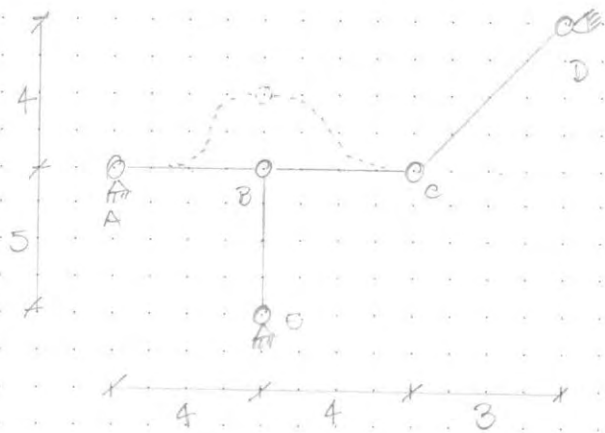
$$\psi_E = -2 \times 10^{-2}$$

$$\psi_{BA} = -\psi_{BC} = \frac{-2 \times 10^{-2}}{4} = -5 \times 10^{-3}$$

$$M_{BA} = 3 \times 10^5 \times 5 \times 10^{-3} = 1500 \text{ Kg-m}$$

$$M_{BC} = 3 \times 10^5 \times (-5 \times 10^{-3}) = -1500 \text{ Kg-m}$$

1.3 Momentos debido al AL por el At



$$\Delta L_{BE} = 15^{\circ} \times 10^{-5} \times 5 = 7,5 \times 10^{-4}$$

$$\psi_{AB} = -\psi_{BC} = \frac{7,5 \times 10^{-4}}{4} = 1,875 \times 10^{-4}$$

$$M_{BA} = 3 \times 10^5 \times (-1,875 \times 10^{-4}) = -56 \text{ Kg-m}$$

$$M_{BC} = 3 \times 10^5 \times (1,875 \times 10^{-4}) = 56 \text{ Kg-m}$$

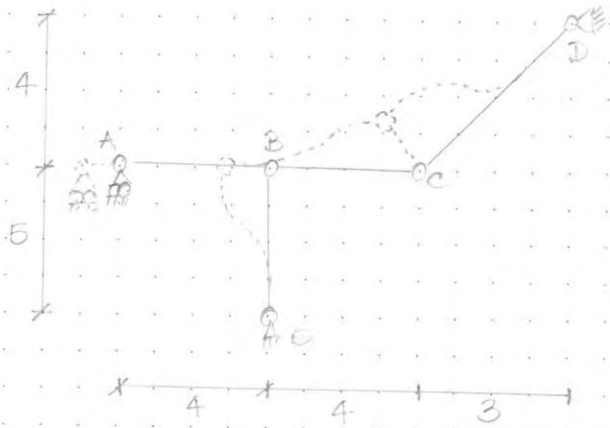
$$M_{EB} = \frac{E K_0 \alpha t (\Delta L_{BE}) \cdot L}{h}$$

$$M_{EB} = \frac{2 \times 10^5 \times 10^{-5} \times (10^8) \times 5}{0,80} = -125$$

$$M_{BE} = 125$$

PREPARADOR: REVISOR: J. V. R. P.

2. Problema con desplazabilidad



$\psi_{EB} = \alpha$

$\delta_B^h = -5\alpha \Rightarrow \psi_{CB} = -\frac{5\alpha}{4}$

$\delta_C^v = \frac{5\alpha}{4} \times 3 = \frac{15\alpha}{4} \Rightarrow \psi_{BC} = \frac{15\alpha}{4} = \frac{15\alpha}{16}$

Cambio de variable, $EK\alpha = 1000x$

$M_{EB} = M_{BE} = -6 \times 2EK\alpha = -12EK\alpha = -12000x$

$M_{BC} = 3EK\alpha \left(-\frac{15}{16}\right) = -\frac{45}{16}EK\alpha = -\frac{45000x}{16}$

$M_{CD} = 3 \times 2EK \times \frac{5\alpha}{4} = \frac{15}{2}EK\alpha = 7500x$

MOMENTOS	M_{EB}	M_{BE}	M_{BC}	M_{CD}	M_{DC}
1.1	-1125	750	-1800	-1500	500
1.2	0	0	1500	-1500	0
1.3	-125	125	-56	56	0
PRIMARIOS	-1250	875	-356	-2944	500

- Factores de Distribución

$S_{BA} = 3EK$

$D_{BA} = 0,214$

$S_{BC} = 3EK$

$D_{BC} = 0,214$

$S_{BD} = \frac{6i}{L} = \frac{6 \times 2EK}{14EK}$

$D_{BD} = 0,572$

PROFESOR REINOSO J. / PÁG. 2

JUNTA	E	B			D
EXT. MIEMBRO	EB	BE	BA	BC	DC
DISTRIBUCIÓN	0	0,572	0,224	0,224	0
TRANSPORTE	0	1/2	0	0	0
PRIMARIOS ρ_D	-1250	875	-356	-2944	500
			-2425		
	694	1387	519	519	
TOTALES ρ_D	-556	2262	163	-2425	500
PRIMARIOS ρ_D	-12000	-12000	0	-45000/26	7500
			-14812,50		
	4236	8473	3170	3170	
TOTALES ρ_D	-7764	-3527	3170	358	7500

- Principio de Trabajo Virtual para despejar "x"

$$\left. \begin{aligned} M_{EB} &= -556 - 7764x \\ M_{BE} &= 2262 - 3527x \end{aligned} \right\} 1706 - 11291x$$

$$M_{BA} = 163 + 3170x$$

$$M_{BC} = -2425 + 358x$$

$$M_{DC} = 500 + 7500x$$

$$(1706 - 11291x)\alpha + (-2425 + 358x) \times \frac{15}{26}\alpha - (500 + 7500x) \times \frac{5}{4}\alpha - 4000 \times \frac{5}{4}\alpha$$

$$- 1000 \times \frac{5}{4}\alpha \times 3 - 3000 \times \frac{15}{26}\alpha + 2000 \times 5\alpha + \frac{900 \times 5}{2} \alpha \times \frac{1}{3} \times 5 = 0$$

$$995,06 - 20330,38x = 0 \Rightarrow x = 0,0489$$

- Momentos Definitivos

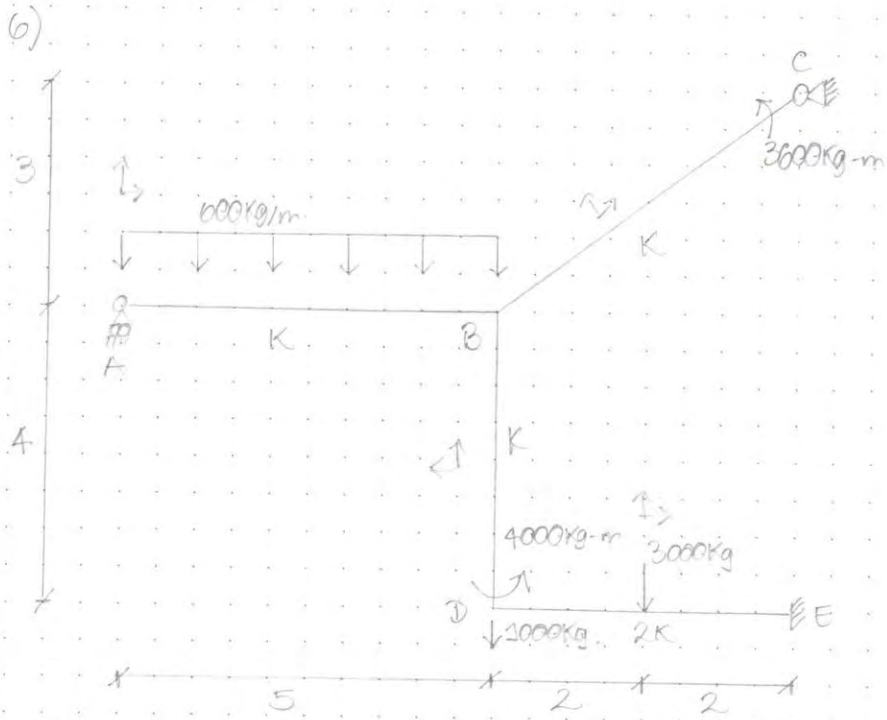
$$M_{EB} = -926 \text{ kg-m}$$

$$M_{BE} = 2030 \text{ kg-m}$$

$$M_{BA} = 318 \text{ kg-m}$$

$$M_{BC} = -2407 \text{ kg-m}$$

$$M_{DC} = 867 \text{ kg-m}$$



$EK = 10^5 \text{ kg-m}$
 $\Delta t_{DE} = 10^\circ\text{C}$
 $\alpha t = 10^{-5} \text{ } ^\circ\text{C}^{-1}$

MÉTODO DE CROSS

1. Problema sin desplazabilidad

1.1. Momentos debido a cargas

$M_{AB}^E = \frac{600 \times 5^2}{12} = 1250 \text{ kg-m}$; $M_{BA}^E = -1250 \text{ kg-m}$

$M_{EA}^E = -1250 + \frac{1}{2}(-1250) = -1875 \text{ kg-m}$

$M_{DE}^E = \frac{1}{2} \times 3000 = 1500 \text{ kg-m}$

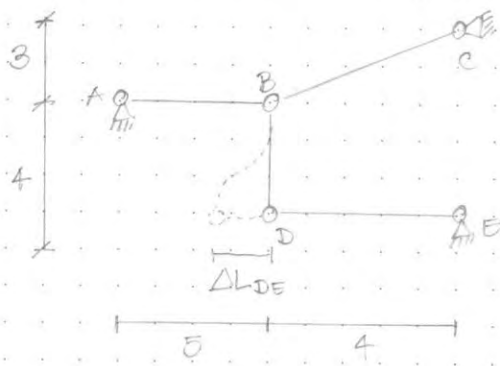
$M_{ED}^E = \frac{3000 \times 4}{8} = 1500 \text{ kg-m}$; $M_{ED}^D = -1500 \text{ kg-m}$

1.2 Momentos debido al Δt por Δt

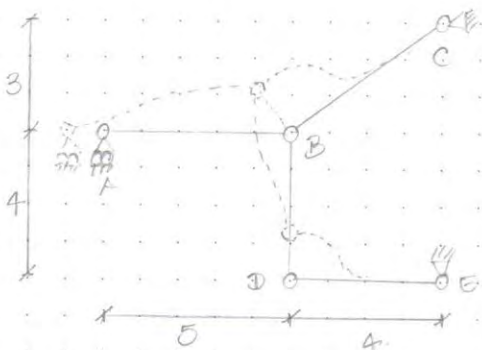
$\Delta L_{DE} = 10^\circ\text{C} \times 10^{-5} \text{ } ^\circ\text{C}^{-1} \times 4 = 4 \times 10^{-4} \text{ m} \Rightarrow \psi_{ED} = -\frac{4 \times 10^{-4}}{4} = -1 \times 10^{-4}$

$M_{ED} = M_{DE} = -6 \times 10^5 \times (-1 \times 10^{-4}) = 60 \text{ kg-m}$

PREPARADOR: RODRIGUEZ S. Y. RUIZ P.



2. Problema con Desplazabilidad



$$P_{DE} = -\alpha$$

$$d_{v_B} = d_{v_D} = 4\alpha \Rightarrow P_{BC} = -\frac{19\alpha}{4} = -\alpha$$

$$P_{AB} = \frac{1}{5}\alpha$$

$$d_{h_D} = 3\alpha \Rightarrow P_{ED} = \frac{3}{4}\alpha$$

Cambio de variable $\Rightarrow \alpha = 1000X$

$$M_{BA} = 30K \times \left(-\frac{1}{5}\right)\alpha = -\frac{12}{5}\alpha EK = -2400X$$

$$M_{BC} = 3EK \times \alpha = 3\alpha EK = 3000X$$

$$M_{ED} = M_{DE} = -6EK \times \frac{3}{4}\alpha = -\frac{18}{4}\alpha EK = -4500X$$

$$M_{DE} = M_{ED} = -6 \times 2EK (-\alpha) = 12\alpha EK = 12000X$$

MOMENTOS	M _{ED}	M _{DE}	M _{BE}	M _{BD}	M _{BA}	M _{BC}
1.1	-1500	1500	0	0	-1875	1800
1.2	0	0	60	60	0	0
PRINCIPALES 90°	-1500	1500	60	60	-1875	1800

PREPARADOR REVISOR I VUELTA

- Factores de Distribución

$$\begin{aligned}
 S_{DB} &= 4EK & I_{DB} &= 0,333 & S_{BA} &= 3EK & D_{BA} &= 0,30 \\
 S_{DE} &= \frac{8EK}{12EK} & D_{DE} &= 0,667 & S_{BC} &= 3EK & I_{BC} &= 0,30 \\
 S_{ED} &= 4EK & & & S_{BD} &= \frac{4EK}{10EK} & D_{BD} &= 0,40
 \end{aligned}$$

JUNTA	E	D		B		
EXT MIEMBRO	ED	DE	DB	EB	BA	BC
DISTRIBUCIÓN	0	0,667	0,333	0,40	0,30	0,30
TRANSPORTE	0	0,50	0,50	0,50	0	0
PRIMARIOS %	-1500	1500	60	60	-1875	1800
		-2440		-15		
	814	1627	813	406		
			-78	-156	-117	-117
	26	52	26	13		
			-2160	-5,20	-3,90	-3,90
	0,86	1,73	0,86			
TOTALES %	-659	3181	813	318	-1996	1679
PRIMARIOS %	12000	12000	-4500	-4500	-2400	3000
		7500		-3900		
	-2501	-5002	-2498	-1249		
			1080	2060	1545	1545
	-343	-686	-343	-172		
			34	69	52	52
	-52	-23	-11	-5,80		
			1,16	2,32	1,74	1,74
TOTALES %	9144	6289	-6287	-3795	-801	4592

- Principio de Trabajo Virtual para despejar "x"

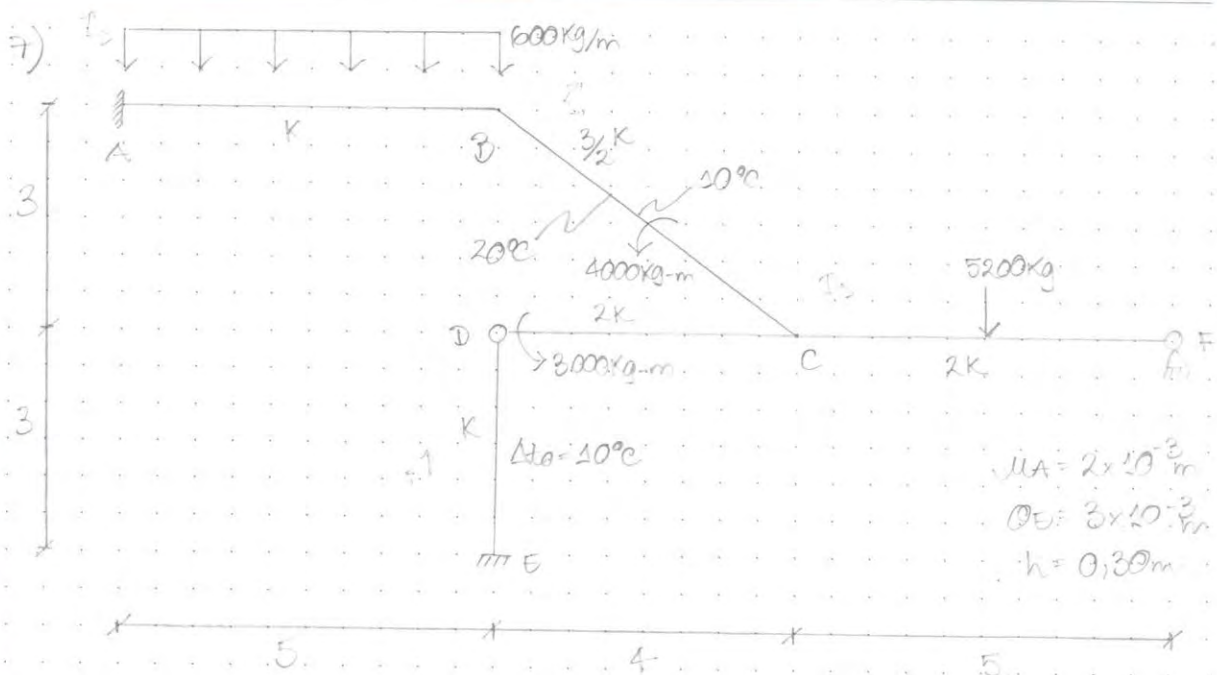
$$\begin{aligned}
 M_{ED} &= -659 + 9144x \\
 M_{DE} &= 3181 + 6289x \\
 M_{DB} &= 813 - 6287x \\
 M_{ED} &= 318 - 3795x \\
 M_{BA} &= -1996 - 801x \\
 M_{BC} &= 1679 + 4592x
 \end{aligned}$$

o Miembros Definitivos

$$\begin{aligned}
 M_{ED} &= -8605 \text{ Kg-m} \\
 M_{DE} &= -2284 \text{ Kg-m} \\
 M_{DB} &= 6282 \text{ Kg-m} \\
 M_{ED} &= 3616 \text{ Kg-m} \\
 M_{BA} &= -1300 \text{ Kg-m} \\
 M_{BC} &= -2328 \text{ Kg-m}
 \end{aligned}$$

$$\begin{aligned}
 &(2522 + 15423x)(-1) + (1187 - 10082x) \times \frac{3}{4} + (-1996 - 801x) \times \frac{1}{6} - (1679 + 4592x) \times \frac{1}{5} - 600 \times 5 \times \frac{4}{5} \times \frac{5}{2} \\
 &- 3600 \times x - 3000 \times 2x - 1000 \times 4x = 0 \quad -24545,05 - 28234,30x = 0 \Rightarrow x = -0,869
 \end{aligned}$$

PREPARADOR: RODRIGUEZ J. VÍCTOR Z.



Método de Cross

- 1- Problema en desplazabilidad
- 1.1. Momentos debido a cargas

$$M_{AB}^E = \frac{qL^2}{12} = \frac{600 \times (5)^2}{12} = 1250 \text{ kg-m} ; M_{BA}^E = -1250 \text{ kg-m}$$

$$M_{BC}^E = \frac{M}{4} = \frac{4000}{4} = 1000 \text{ kg-m} = M_{CB}^E$$

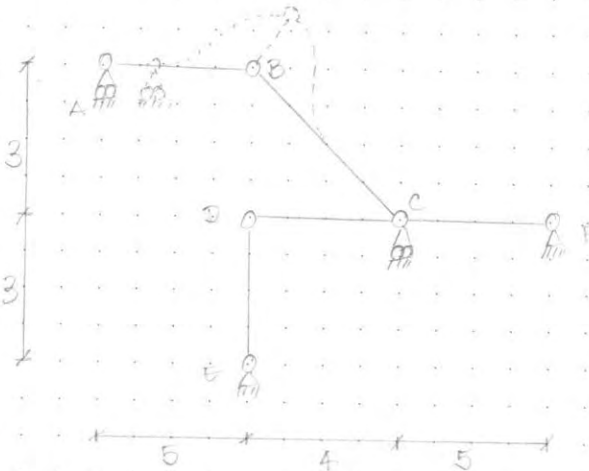
$$M_{CF}^E = \frac{P \times L}{8} = \frac{5200 \times 5}{8} = 3250 \text{ kg-m} ; M_{FC}^E = -3250 \text{ kg-m}$$

$$M_{CD}^* = M_{CF}^E + \frac{1}{2} (M_{BC}^E + M_{CB}^E) = 3250 + \frac{1}{2} (1000 - (-3250)) = 4975 \text{ kg-m}$$

$$M_{DC}^* = \frac{1}{2} (3000) = 1500 \text{ kg-m}$$

PREPARADOR PEDUELA J. V. P. 12

1.2 Momentos debido a los movimientos de apoyo.



$$\psi_{BC} = \frac{2 \times 10^{-3}}{3}$$

$$\delta_B^y = \frac{2 \times 10^{-3} \times 4}{3} = \frac{8}{3} \times 10^{-3}$$

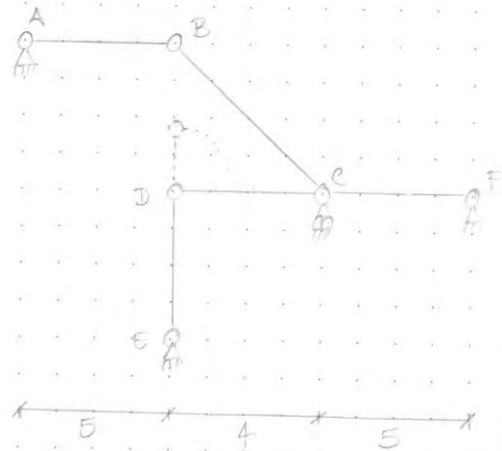
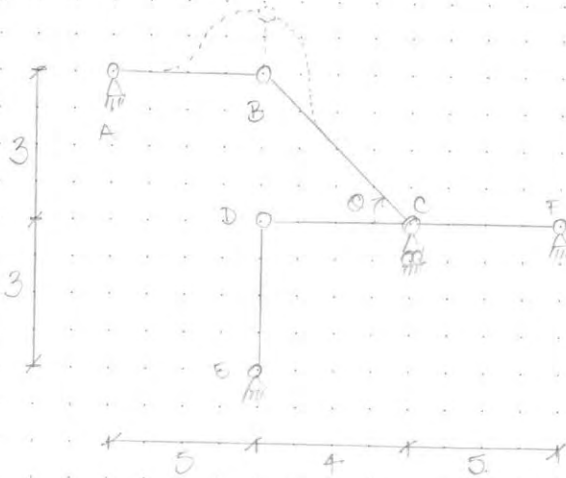
$$\psi_{AB} = \frac{\frac{8}{3} \times 10^{-3}}{5} = \frac{8}{15} \times 10^{-3}$$

$$M_{AB} = M_{BA} = -6 \times 10^5 \times \frac{8}{15} \times 10^{-3} = -320 \text{ kg-m}$$

$$M_{BC} = M_{CB} = -6 \times \frac{3}{2} \times 10^5 \times \frac{8}{3} \times 10^{-3} = -600 \text{ kg-m}$$

$$M_{CD} = 3 \times 10^5 \times 3 \times 10^{-3} = 900 \text{ kg-m}$$

1.3 Momentos debido al ΔI por Δt



$$\Delta I_{BC} = 15^\circ \times 10^{-5} \times 5 = 7,5 \times 10^{-4}$$



$$\psi_{BC} = \frac{3}{5} = \frac{7,5 \times 10^{-4}}{\delta_B^y}$$

$$\delta_B^y = 1,25 \times 10^{-3}$$

$$\psi_{AB} = \frac{1,25 \times 10^{-3}}{5} = 2,5 \times 10^{-4}$$

$$\Delta I_{DE} = 10^\circ \times 10^{-5} \times \frac{3}{2} = 3 \times 10^{-4}$$

$$\psi_{DE} = \frac{-3 \times 10^{-4}}{4} = -7,5 \times 10^{-5}$$

$$tg \theta = \frac{7,5 \times 10^{-4}}{y} = \frac{3}{4} \Rightarrow y = 1 \times 10^{-3} \text{ m} \Rightarrow \psi_{bc} = -\frac{1 \times 10^{-3}}{5} = -2 \times 10^{-4}$$

$$M_{AB} = M_{BA} = -6 \times 10^5 \times 2,5 \times 10^{-4} = -150 \text{ kg-m}$$

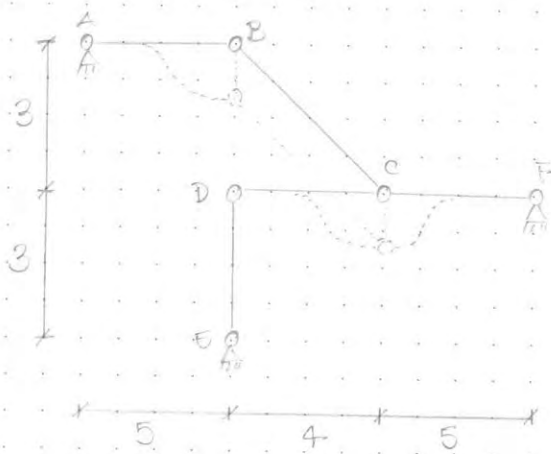
$$M_{bc} = M_{cb} = -6 \times \frac{3}{2} \times 10^5 \times (2 \times 10^{-4}) = 180 \text{ kg-m}$$

$$M_{bc} = -M_{cb} = \frac{3}{2} \times 10^5 \times 10^{-5} \times 10 \times 5$$

$$M_{cd} = 3 \times 2 \times 10^5 \times 7,5 \times 10^{-5} = 45 \text{ kg-m}$$

$$0,30 = 250 \text{ kg-m}$$

2- Problema con desplazabilidad



$$\psi_{AB} = -\alpha$$

$$d_c^v = 5\alpha \Rightarrow \psi_{cd} = -\frac{5}{4}\alpha \text{ y } \psi_{cf} = \alpha$$

$$M_{AB} = M_{BA} = -6EK(-\alpha) = 6EK\alpha = 6000\alpha$$

$$M_{cd} = 3 \times 7EK \times \frac{5}{4} \alpha = \frac{30}{4} EK\alpha = 7500\alpha$$

$$M_{cf} = 3 \times 2EK(-\alpha) = -6EK\alpha = -6000\alpha$$

MOMENTOS	M _{AB}	M _{BA}	M _{bc}	M _{cb}	M _{cf}	M _{cd}	M _{ed}
1.1	1250	-1250	1000	1000	4875	1500	0
1.2	-320	-320	600	600	0	0	200
1.3	-150	-150	480	-70	0	45	0
PRIMARIOS	780	-1720	2080	1530	4875	1545	200

- Factores de Distribución

$$D_{BA} = 4EK$$

$$D_{BA} = 0,40$$

$$D_{CB} = 4 \times \frac{3}{2} EK = 6EK$$

$$D_{CB} = \frac{1}{2}$$

$$D_{bc} = 4 \times \frac{3}{2} EK = 6EK$$

$$D_{bc} = 0,60$$

$$D_{cf} = 3 \times 2EK = 6EK$$

$$D_{cf} = \frac{1}{3}$$

$$D_{cd} = 3 \times 2EK = 6EK$$

$$D_{cd} = \frac{1}{3}$$

JUNTA	A	B		C			E
EXT MIEMBRO	AB	BA	BC	CB	CF	CD	ED
DISTRIBUCIÓN	0	0,40	0,60	1/2	1/3	1/3	0
TRANSPORTE	0	1/2	1/2	1/2	0	0	0
PRIMARIES %	7800	-1720	2080	5580	4875	1545	900
		310		7950			
			-1325	-2650	-2650	-2650	
	203	406	609	805			
			-51	-10250	-10250	-10250	
	10	20	31	1550			
			-2160	-5126	-5126	-5126	
		104	155				
TOTALES %	993	-1293	1293	-906	2110	-1212	900
PRIMARIES %	6000	6000	0	0	-6000	7500	0
		6000		1500			
	-1200	-2400	-3600	-1800			
			50	100	100	100	
	-10	-20	-30	-15			
			250	5	5	5	
	-0,50	-1	-1,50				
TOTALES %	4720	2572	-3572	-1720	-5895	7605	0

- Principio de Trabajo Virtual para despegar "x"

- Momentos Definitivos

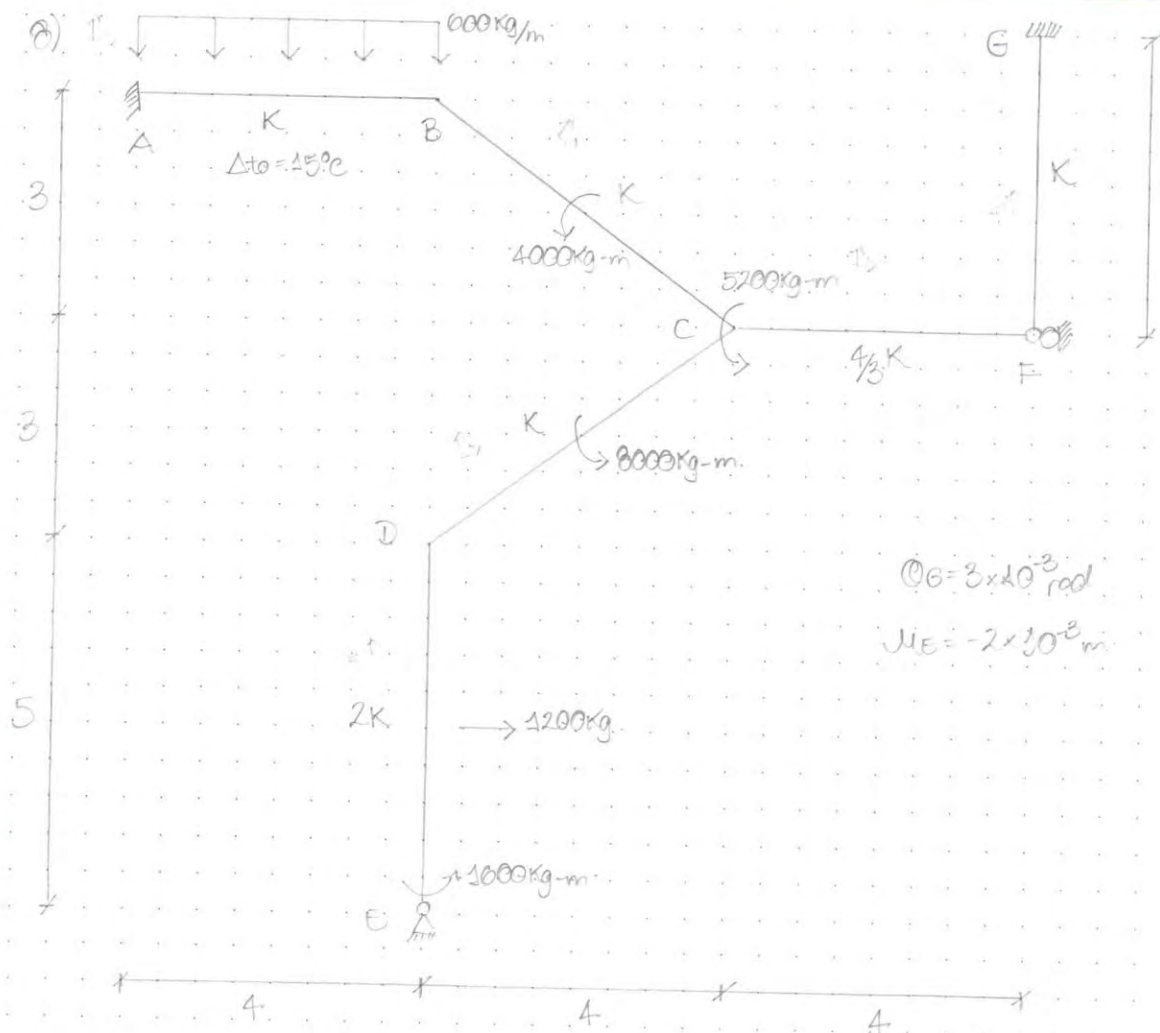
$$\begin{aligned}
 M_{AB} &= 993 + 4720x \\
 M_{BA} &= -1293 + 2572x \\
 M_{BC} &= -1293 - 3572x \\
 M_{CB} &= -906 - 1720x \\
 M_{CF} &= 2110 - 5895x \\
 M_{CD} &= -1212 + 7605x \\
 M_{ED} &= 900 \text{ kg-m}
 \end{aligned}$$

$$\begin{aligned}
 M_{AB} &= 5100 \text{ kg-m} \\
 M_{BA} &= 1821 \text{ kg-m} \\
 M_{BC} &= -1821 \text{ kg-m} \\
 M_{CB} &= -2324 \text{ kg-m} \\
 M_{CF} &= -3011 \text{ kg-m} \\
 M_{CD} &= 5404 \text{ kg-m} \\
 M_{ED} &= 900 \text{ kg-m}
 \end{aligned}$$

$$(-300 + 8269x) \cdot (-\alpha) + (2110 - 5895x) \cdot \alpha - (-1212 + 7605x) \cdot \frac{5}{4} \alpha + 600 \times 5 \times \frac{5}{2} \alpha + 5200 \times \frac{5}{2} \alpha - 3000 \times \frac{5}{4} \alpha = 0$$

$$20683 - 23770,25x = 0 \Rightarrow x = 0,870$$

PREPARADO POR VIRIL R



$\Theta_G = 3 \times 10^{-3} \text{ rad}$
 $\Psi_E = -2 \times 10^{-2} \text{ m}$

MÉTODO DE CROSS

1.- Problema sin desplazabilidad

1.1. Momentos debido a cargas

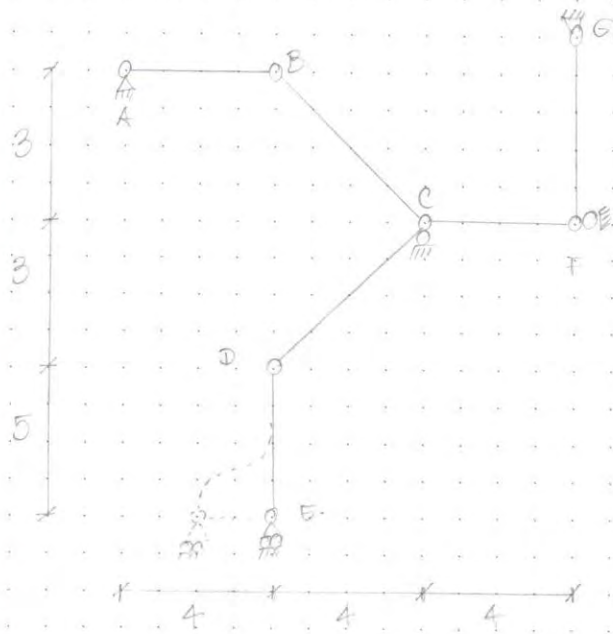
$M_{AB} = -M_{BA} = \frac{q \times l^2}{12} = \frac{600 \times (4)^2}{12} = 800 \text{ kg-m}$

$M_{BC} = M_{CB} = \frac{H}{4} = 5000 \text{ kg-m}$; $M_{CD} = M_{DC} = 2000 \text{ kg-m}$

$M_{ED}^E = -M_{DE}^E = \frac{1200 \times 5}{6} = 750 \text{ kg-m}$; $M_{DE}^* = -750 + \frac{1}{2} (1600 - 750) = -325 \text{ kg-m}$

PREPARADOR: HÉCTOR J. VILLO R.

1.2 Momentos debido a Movimientos de Argo

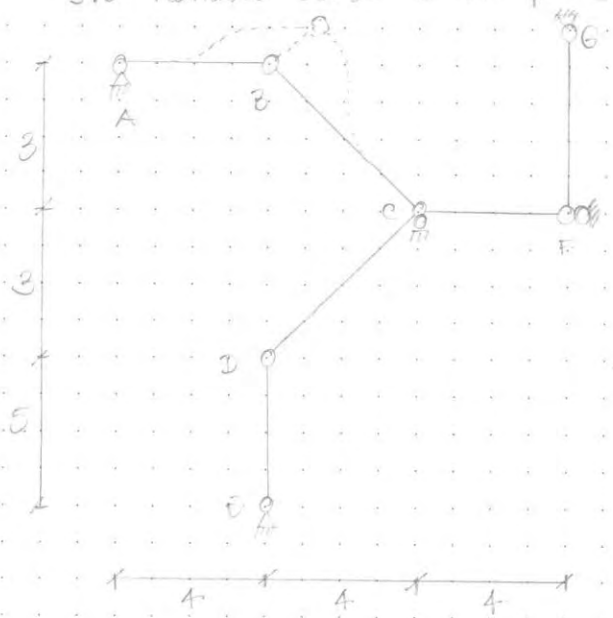


$$i_{BC} = \frac{-2 \times 10^{-3}}{5} = -4 \times 10^{-4}$$

$$M_{DE} = 3 \times 2 \times 10^5 \times (4 \times 10^{-4}) = 240 \text{ Kg-m}$$

$$M_{EF} = 3 \times 10^5 \times 3 \times 10^{-3} = 900 \text{ Kg-m}$$

1.3 Momentos debido al Al por el Av



$$AL_{AB} = 150 \times 10^{-5} \times 10^{-4} \times 4 = 6 \times 10^{-4}$$

$$V_{BC} = -\frac{6 \times 10^{-4}}{3} = -2 \times 10^{-4}$$

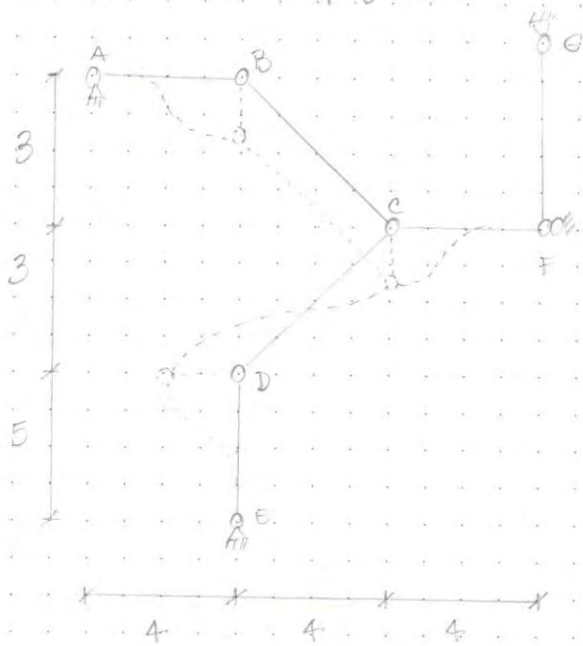
$$d_B^V = 2 \times 10^{-4} \times 4 = 8 \times 10^{-4}$$

$$V_{AB} = \frac{8 \times 10^{-4}}{4} = 2 \times 10^{-4}$$

$$M_{AB} = M_{BA} = -6 \times 10^5 \times 2 \times 10^{-4} = -120 \text{ Kg-m}$$

$$M_{BC} = M_{CB} = -6 \times 10^5 \times (-2 \times 10^{-4}) = 120 \text{ Kg-m}$$

2. Problema con desplazabilidad



$\psi_{EF} = \alpha$

$\delta_C^V = 4\alpha \Rightarrow \psi_{AB} = -\frac{4\alpha}{4} = -\alpha$

$\psi_{CD} = -\frac{4\alpha}{4} = -\alpha$

$\delta_D^H = 3\alpha \Rightarrow \psi_{DE} = \frac{3\alpha}{5}$

$M_{AB} = M_{BA} = -6EK(-\alpha) = 6EK\alpha = 6000X$

$M_{CF} = 3 \times \frac{4}{3} EK(-\alpha) = -4EK\alpha = -4000X$

$M_{CD} = M_{DC} = -6EK(-\alpha) = 6EK\alpha = 6000X$

$M_{DE} = 2 \times 2EK(-\frac{3\alpha}{5}) = -\frac{12}{5} EK\alpha = -2400X$

MOMENTOS	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CF}	M_{CD}	M_{DC}	M_{DE}	M_{EF}
1.1	800	-800	1000	1000	0	2000	2000	-325	0
1.2	0	0	0	0	0	0	0	240	300
1.3	-120	-120	120	120	0	0	0	0	0
FRUSTRACIONES	600	-920	1120	1120	0	2000	2000	-85	300

- Factores de Distribución

$S_{BA} = 4EK$

$D_{BA} = 0/50$

$S_{CB} = 4EK$

$D_{CB} = 1/3$

$S_{BC} = \frac{4EK}{3}$

$D_{BC} = 0/50$

$S_{CF} = 3 \times \frac{4}{3} EK = 4EK$

$D_{CF} = 1/3$

$S_{CD} = \frac{4EK}{12EK}$

$D_{CD} = 1/3$

$S_{DE} = 4EK$

$D_{DE} = 0/40$

$S_{FE} = 2 \times 2EK = \frac{6EK}{10EK}$

$D_{FE} = 0/60$

PREPARADOR: REDESAR I. VIRAL R.

JUNTA	A	B		C			D		G
EXT MIEMBRO	AB	BA	BC	CB	CF	CD	DC	DE	EG
DISTRIBUCIÓN	0	0,50	0,50	1/3	1/3	1/3	0,40	0,60	0
TRANSPORTE	0	1/2	1/2	1/2	0	1/2	1/2	0	0
PRIMARIOS %	6000	-220	1120	1120	0	2000	2000	-85	200
		200		-2080			1215		
		247	623,20	623,20	623,20	247			
	-137	-274	-274	-137		-453	-205	-1357	
		2840	28670	28670	28670	28670	28670	28670	
	-2460	-4270	-4270	-2460		-3250	-32	-52	
		725	1470	1470	1470	725			
	-184	-368	-368	-184		-147	-224	-441	
		0,55	1,10	1,10	1,10	0,55			
TOTALES %	517	-1247	1247	1802	206	2432	1506	-1506	200
PRIMARIOS %	6000	6000	0	0	-4000	6000	6000	-3600	0
		6000		2000			2400		
	-1500	-2000	-3000	-1500		-600	-1200	-1200	
		16,66	33,33	33,33	33,33	16,66			
	-416	-833	-833	-416		-3,33	-6,66	-9,99	
		1,25	2,50	2,50	2,50	1,25			
		-0,62	-0,62			-0,50	-0,75		
TOTALES %	4426	2991	-2991	-1468	-3264	5432	4811	-4811	0

- Principio de Trabajo Virtual para despejar "x"

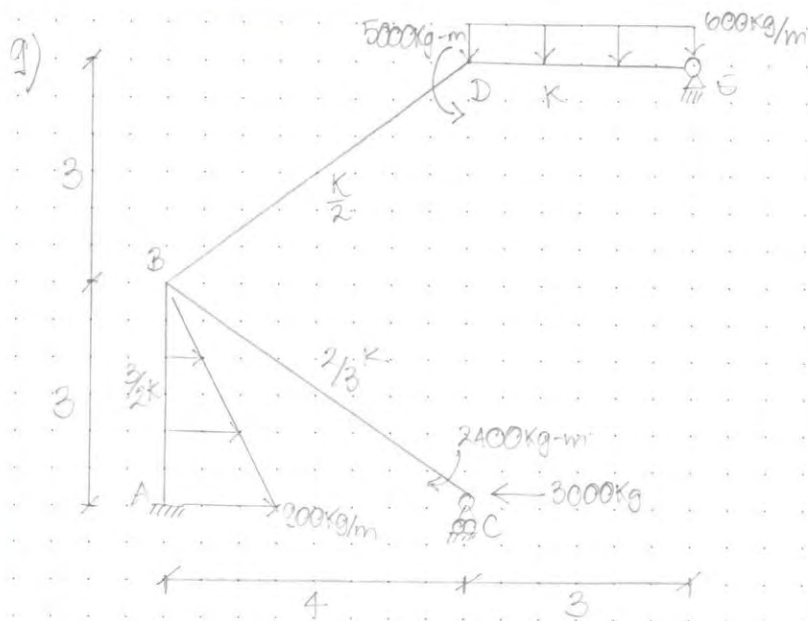
$$\begin{aligned}
 M_{AB} &= 517 + 4426x \\
 M_{BA} &= -1247 + 2991x \\
 M_{BC} &= 1247 - 2991x \\
 M_{CB} &= 1802 - 1468x \\
 M_{CF} &= 206 - 3264x \\
 M_{CD} &= 2432 + 5432x \\
 M_{DC} &= 1506 + 4811x \\
 M_{DE} &= -1506 - 4811x \\
 M_{EG} &= 200 \text{ kg-m}
 \end{aligned}$$

- Momentos Indefinidos

$$\begin{aligned}
 M_{AB} &= -809 \text{ kg-m} \\
 M_{BA} &= -2129 \text{ kg-m} \\
 M_{BC} &= 2129 \text{ kg-m} \\
 M_{CB} &= 2295 \text{ kg-m} \\
 M_{CF} &= 207,5 \text{ kg-m} \\
 M_{CD} &= 830 \text{ kg-m} \\
 M_{DC} &= 87 \text{ kg-m} \\
 M_{DE} &= -87 \text{ kg-m} \\
 M_{EG} &= 200 \text{ kg-m}
 \end{aligned}$$

$$\begin{aligned}
 &(-730 + 7487x)(L-d) + (32938 + 10243x)(L-d) + (206 - 3264x) \cdot d + (-1506 - 4811x) \cdot \frac{3d}{5} + 600 \cdot 4x \cdot 2d \\
 &- 8000x - 1200 \times \frac{3}{5} d \times \frac{5}{2} + 1600 \times \frac{3}{5} d = 0 \quad -7245,60 - 24583,60x = 0 \\
 &x = -0,295
 \end{aligned}$$

PREPARADOR PEDRO J. VIREL R.



$$\sqrt{A} = -2 \times 10^{-2} \text{ m}$$

$$\Delta t_{0.25} = 10 \text{ pc}$$

MÉTODO DE CROSS

1. Problema en desplazabilidad

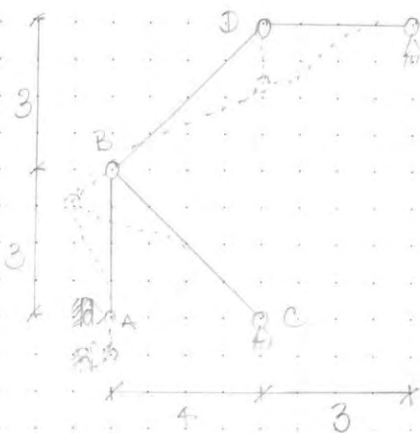
1.1. Momentos debidos a cargas

$$M_{AB}^E = \frac{q \times l^3}{20} = \frac{200 \times (3)^3}{20} = 405 \text{ kg-m} \quad ; \quad M_{BA}^E = -\frac{q l^3}{30} = -\frac{200 \times (3)^3}{30} = -270 \text{ kg-m}$$

$$M_{BC}^E = \frac{1}{2} \times (-2400) = -1200 \text{ kg-m}$$

$$M_{DE}^E = -M_{ED}^E = \frac{q \times l^2}{12} = \frac{600 \times (3)^2}{12} = 450 \text{ kg-m} \Rightarrow M_{DE}^E = 450 + \frac{1}{2} \times (-450) = 675 \text{ kg-m}$$

1.2. Momentos debidos a movimientos de apoyo



$$\psi_{BC} = \frac{2 \times 10^{-2}}{4} = 5 \times 10^{-3} \quad \delta_B^h = 5 \times 10^{-3} \times 3 = 0,015$$

$$\psi_{AB} = \frac{0,015}{3} = 5 \times 10^{-3} \quad \psi_{ED} = \frac{0,015}{3} = -5 \times 10^{-3}$$

$$\delta_D^v = 5 \times 10^{-3} \times 4 = 0,02 \Rightarrow \psi_{DC} = \frac{0,02}{3} = 6,667 \times 10^{-3}$$

$$M_{AB} = M_{BA} = -6 \times \frac{10^5}{2} \times 5 \times 10^{-3} = -4500$$

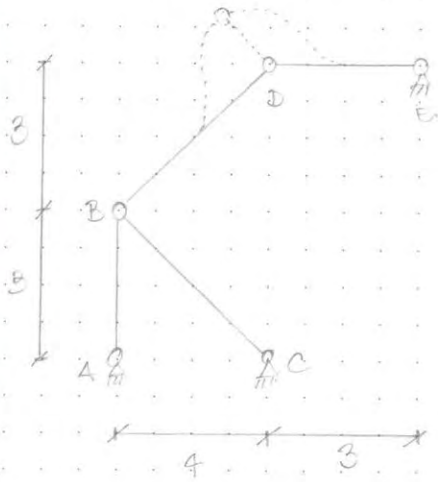
$$M_{ED} = M_{DE} = -6 \times \frac{10^5}{2} \times (-5 \times 10^{-3}) = 4500$$

PREPARADOR PEDROAR J. VARELA

$$M_{BC} = 3 \times \frac{2}{3} \times 10^5 \times (-5 \times 10^{-3}) = -1000 \text{ kg-m}$$

$$M_{DE} = 3 \times 10^5 \times (-6,667 \times 10^{-3}) = -2000 \text{ kg-m}$$

1.3. Momentos debido al ΔL por Δt .



$$\Delta L_{DE} = 10^5 \times 10^{-5} \times 3 = 3 \times 10^{-4} \text{ m}$$

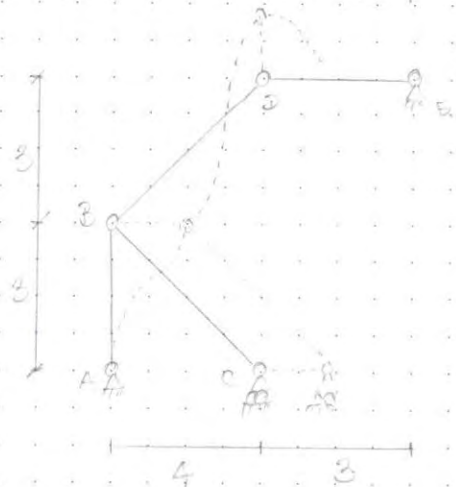
$$\psi_{ED} = \frac{3 \times 10^{-4}}{3} = 1 \times 10^{-4}$$

$$M_{ED} = M_{DE} = -6 \times \frac{10^5}{2} \times 1 \times 10^{-4} = -30 \text{ kg-m}$$

$$\delta_D^y = 1 \times 10^{-4} \times 4 = 4 \times 10^{-4} \Rightarrow \psi_{DE} = \frac{4}{3} \times 10^{-4}$$

$$M_{DE} = 3 \times 10^5 \times \left(\frac{4}{3} \times 10^{-4}\right) = 40 \text{ kg-m}$$

2. Problema con desplazabilidad



$$\psi_{AB} = -\alpha$$

$$\delta_B^h = 3\alpha \Rightarrow \psi_{ED} = \frac{3\alpha}{3} = \alpha$$

$$\delta_D^y = 4\alpha \Rightarrow \psi_{DE} = -\frac{4}{3}\alpha$$

$$M_{AB} = M_{BA} = -6 \times \frac{3EK}{2} (-\alpha) = 9EK\alpha = 9000\alpha$$

$$M_{ED} = M_{DE} = -6 \times \frac{3EK}{2} \times \alpha = -9EK\alpha = -9000\alpha$$

$$M_{DE} = 3EK \times \frac{4}{3}\alpha = 4EK\alpha = 4000\alpha$$

- Factores de Distribución

$$S_{BA} = 4 \times \frac{3EK}{2} = 6EK \quad D_{BA} = 0,60$$

$$S_{DB} = 4 \times \frac{EK}{2} = 2EK \quad D_{DB} = 0,4$$

$$S_{BC} = 3 \times \frac{2EK}{3} = 2EK \quad D_{BC} = 0,20$$

$$S_{DE} = \frac{3EK}{3EK} \quad D_{DE} = 0,60$$

$$S_{BE} = 4 \times \frac{EK}{2} = 2EK \quad D_{BE} = 0,20$$

$$10EK$$

MOMENTOS	MAB	MBA	MBC	MBD	MDB	MDE
1.1	405	-270	-1200	0	0	675
1.2	-4500	-4500	-1000	1500	1500	-2000
1.3	0	0	0	-30	-30	40
PRIMARIOS %	-4095	-4770	-2200	1470	1470	-1285

JUNTA	AB	BA	BC	BD	DB	DC
EXT. MEMBRO						
DISTRIBUCIÓN	0	0,60	0,20	0,20	0,40	0,60
TRANSPORTE	0	1/2	0	1/2	1/2	0
PRIMARIOS %	-4095	-4770	-2200	1470	1470	-1285
			-5500		-4015	
	1650	3200	1100	1100	550	
				853	1706	2559
	-25580	-51180	-17060	-17060	-8530	
				1705	3410	5120
	-5152	-10231	-3441	-3441	-1721	
					0,68	1,03
TOTALES %	-2706	-1992	-1274	3266	3674	1326
PRIMARIOS %	3000	3000	0	-3000	-3000	4000
			6000		1000	
	-1800	-3600	-1200	-1200	-600	
				-80	-160	-240
	24	48	16	16	8	
				-160	-320	-480
		0,96	0,32	0,32		
TOTALES %	7224	5449	-1184	-4265	-3755	3755

-Paro de Trabajo Virtual para despejar "x"

$$M_{AB} = -2706 + 7224x \quad ; \quad -4698 + 12673x \quad ; \quad M_{BD} = 3266 - 4265x \quad ; \quad 6940 - 8020x$$

$$M_{BA} = -1992 + 5449x \quad ; \quad M_{DB} = 3674 - 3755x$$

$$M_{BC} = -1274 - 1184x \quad ; \quad M_{DC} = 1326 + 3755x$$

PREPARADOR REDONDO J. VIREL P.

Proyecto TEMA 4 - MÉTODO DE CROSS

Fecha OCTUBRE 2012

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$$(-4698 + 12673x) \cdot (1-x) + (6940 - 8020x) \cdot (x) + (1326 + 3755x) \cdot \left(-\frac{4}{3}x\right) - 600 \times 3 \times \frac{4}{3} \times \frac{3}{2} - 3000 \times 3x + \frac{900 \times 3}{2} \times x = 0$$

$$-1380 - 25700x = 0 \Rightarrow x = -0,054$$

• Momentos Definitivos

$$M_{AB} = -3096 \text{ Kg-m}$$

$$M_{BA} = -2286 \text{ Kg-m}$$

$$M_{BC} = -1210 \text{ Kg-m}$$

$$M_{BD} = 3496 \text{ Kg-m}$$

$$M_{DB} = 3877 \text{ Kg-m}$$

$$M_{DE} = 1123 \text{ Kg-m}$$

PREPARADOR: REYESCAR J. VIREL R.