ESTEBAN TENREIRO & Profesionales asociados

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Piso 1, oficina 104, Santa Rosa de Lima
Caracas 1060 - Venezuela
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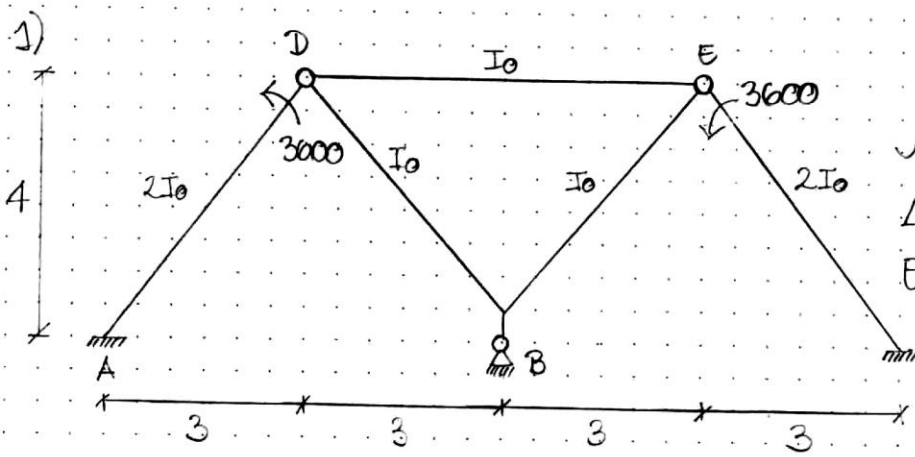
1)



$\mu_B = 3 \times 10^{-2}$ m

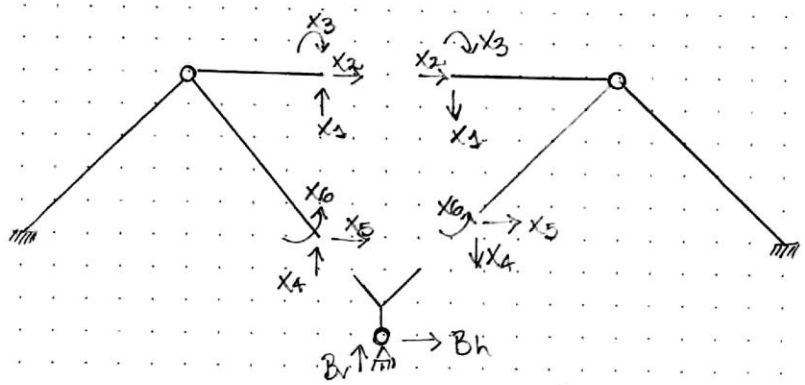
$\Delta t_o^{AD} = -\Delta t_o^{EC} = 10°C$

$EI_o = 10^6 Kg \cdot m^2$

*ESTRUCTURA SIMÉTRICA CON CARGAS ANTISIMÉTRICAS

Nótese que el cambio de temperatura DILATA a la barra AD, mientras que CONTRAE a la barra EC (su par simétrico).

Se plantea el corte de la estructura:



X_1 ∃ ✓

$X_2 = 0$

$X_3 = 0$

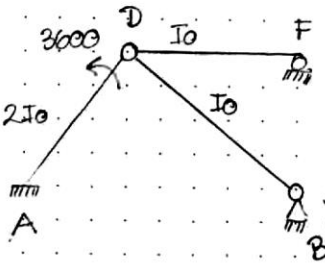
X_4 ∃ ✓

X_5 ∃ ✓

$X_6 = 0$

PREPARADOR: REDEGAR J VIREL R

El sistema equivale a:



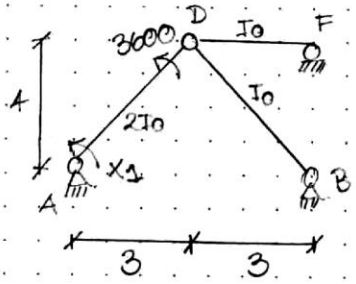
$EIo = 10^6 Kg-m^2$ Sistema Hiperestático de Grado 1.

$\Delta to^{AD} = 10°C$

$\mu_B = 3 \times 10^{-2} m$

- MÉTODO DE LAS FUERZAS

-Sistema Primario



Ecuación de Compatibilidad

$D_1 = 0$

El trabajo realizado por la redundante "X_1" es cero porque el extremo de miembro "A" está empotrado a Tierra y no experimenta rotación

-Sistema Cero

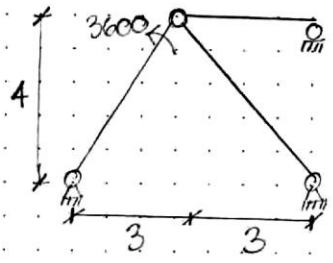
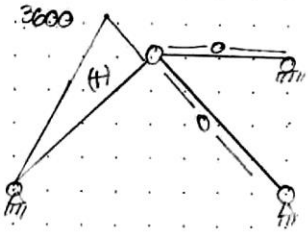


DIAGRAMA DE MOMENTOS



-Sistema Uno

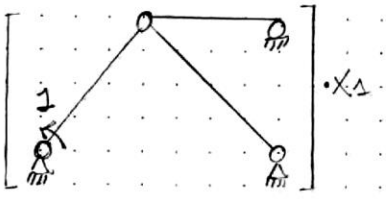
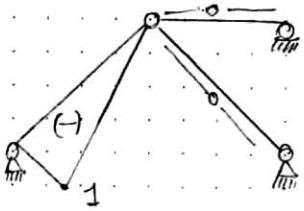


DIAGRAMA DE MOMENTOS

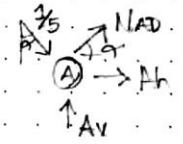


PREPARADOR: REDESCAR J VIREL R

$\sum \overset{\curvearrowleft}{M_B} = 0 \quad -Av \times 6 + 1 = 0 \Rightarrow Av = 1/6$

$\sum \overset{\curvearrowleft}{M_{AD}} = 0 \quad 1 - Av \times 3 + 4Ah = 0 \Rightarrow Ah = -1/8$

NODO "A"



$\sum \overset{+\rightarrow}{Fh} = 0 \quad NAD \times \cos\alpha + \frac{1}{5} \times \operatorname{sen}\alpha + Ah = 0$

$NAD \times \frac{3}{5} + \frac{1}{5} \times \frac{4}{5} - \frac{1}{8} = 0 \Rightarrow NAD = \frac{-7}{120}$

$\sum \overset{\curvearrowleft}{M_F} = 0 \quad Bh \times 4 + 1 - Av \times 6 + Ah \times 4 = 0 \Rightarrow Bh = \frac{1}{8}$

Por superposición se tiene que

$$D_1 = D_{10} + d_{11} \cdot X_1$$

Donde $D_1 = 0$.

$$\underbrace{D_{10} + \frac{1}{8} \times 3 \times 10^2 \times \frac{10^6}{EI_0}}_{P \times M_B} = \underbrace{\frac{(-1) \times 3600 \times 5}{6 \times 2EI_0}}_{\int \frac{\bar{M} M}{EI_0} ds} + \underbrace{10^{-5} \times 10 \times \left(\frac{-7}{120} \times 5\right) \times \frac{10^6}{EI_0}}_{\alpha t \times \Delta t_0 \times \int \bar{N} \cdot ds}$$

$$D_{10} = \frac{-31675}{6EI_0}$$

$$d_{11} = \frac{(-1) \times (-1) \times 5}{3 \times 2EI_0} = \frac{5}{6EI_0}$$

$$0 = \frac{-31675}{6EI_0} + \frac{5}{6EI_0} \times X_1$$

$$\boxed{X_1 = 6335 \text{ Kg-m}}$$

PREPARADOR: REDESCAR J. VIRUET R.

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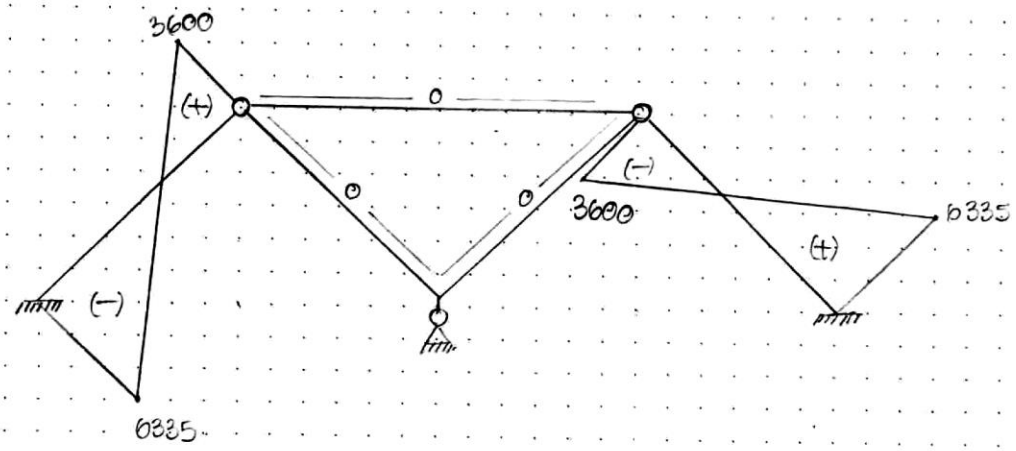
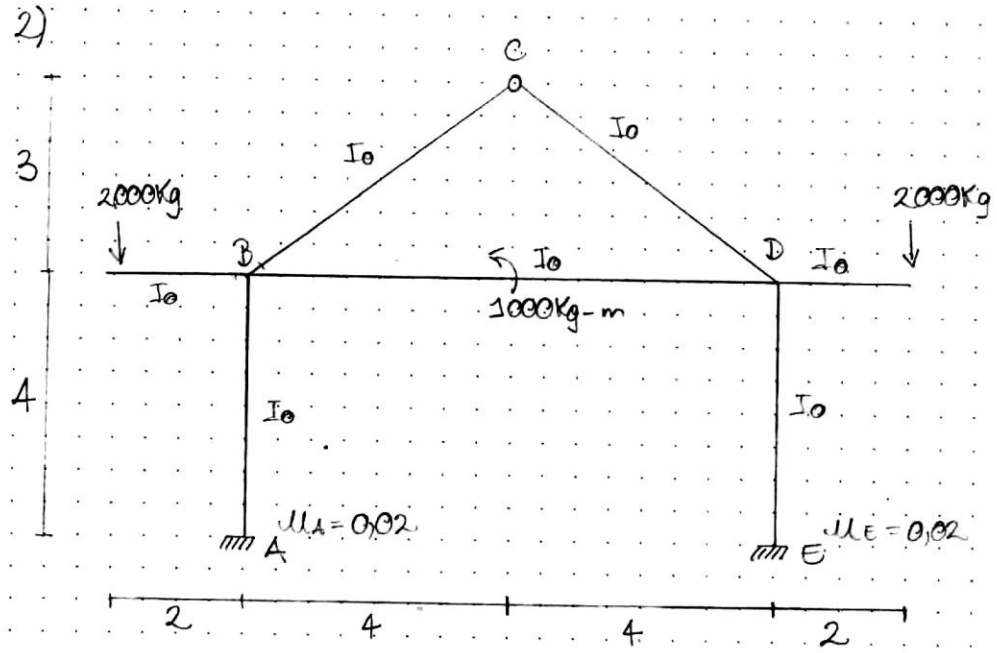
ODIAGRAMAS FINALES.



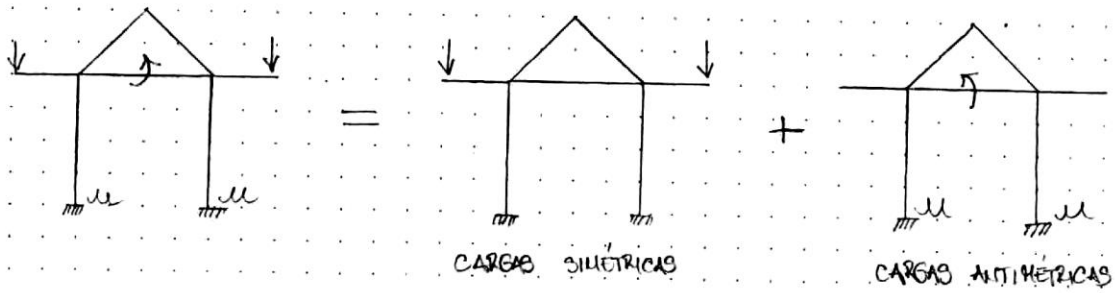
PREPARADOR: REDESCAR J. V/REL 2

2)



ESTRUCTURA SIMÉTRICA CON CARGAS SIMÉTRICAS Y CARGAS ANTIMÉTRICAS.

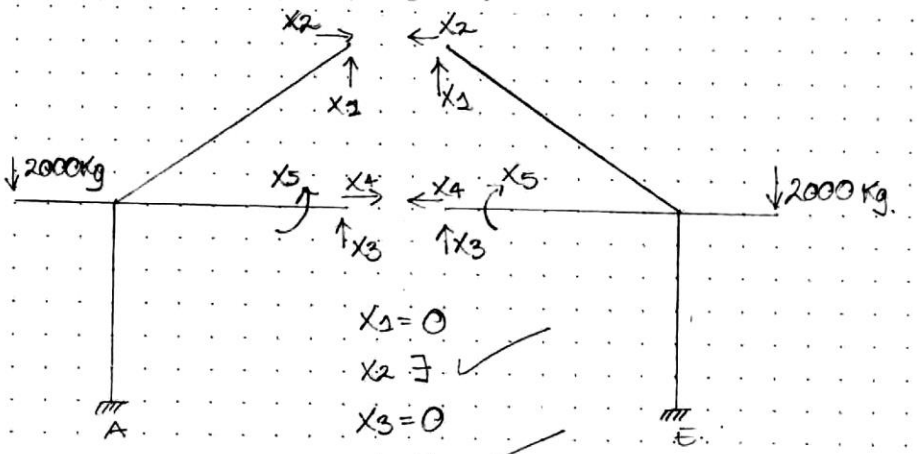
Se debe subdividir el problema en dos partes, una tratando el caso simétrico de cargas, y la otra resolviendo el sistema antimétrico de cargas.



PREPARADOR: REDESCAR J. VIRGEL R.

°ESTRUCTURA SIMÉTRICA CON CARGAS SIMÉTRICAS

-Se plantea el corte de la estructura:



$X_1 = 0$

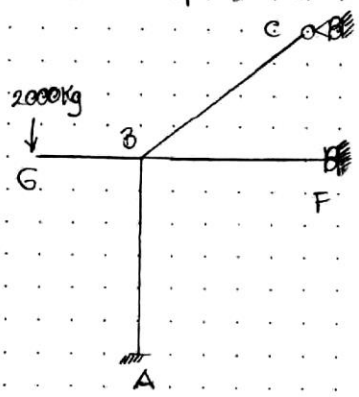
X_2 ∃ ✓

$X_3 = 0$

X_4 ∃ ✓

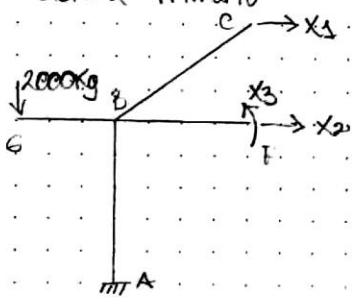
X_5 ∃ ✓

-El sistema equivale a:



° MÉTODO DE LAS FUERZAS

-Sistema Primario



Ec de compatibilidad

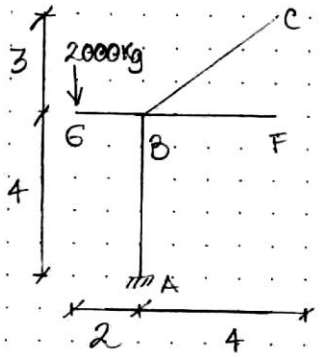
El trabajo realizado por las fuerzas internas X_1, X_2 y X_3 es cero

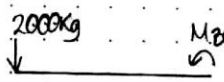
$D_1 = D_2 = D_3 = 0$.

PREPARADOR: REDECEAR J. VIREL R.

PREPARADOR: REINOSO J. VIRGIL R.

-Sistema Cero.





$\sum M_B^{\curvearrowleft} = 0$
BG

$M_{BG} + 2000 \times 2 = 0$

$M_{BG} = -4000 Kg\text{-}m$

$\sum M_A^{\curvearrowleft} = 0$

$-M_A + 2000 \times 2 = 0$

$M_A = 4000 Kg\text{-}m$

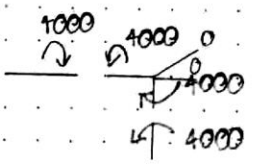
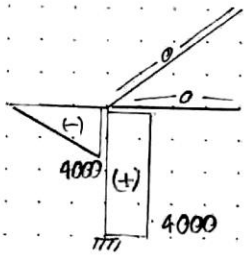
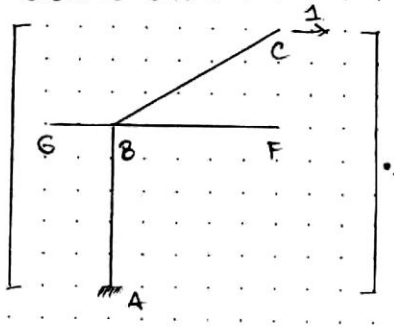


DIAGRAMA DE MOMENTOS



-Sistema Uno



$\sum M_B^{\curvearrowleft} = 0$
BC

$-M_{BC} - 1 \times 3 = 0$

$M_{BC} = -3$

$\sum M_A^{\curvearrowleft} = 0$

$-1 \times 7 - M_A = 0$

$M_A = -7$

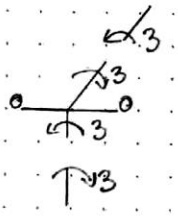
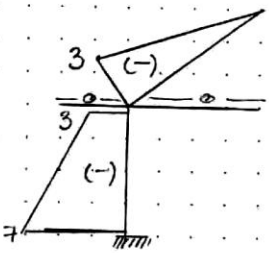
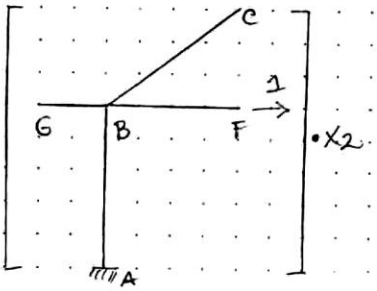


DIAGRAMA DE MOMENTOS



-Sistema Dos.

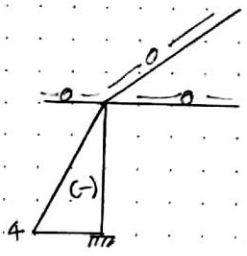


$\sum M_A^{\curvearrowleft} = 0$

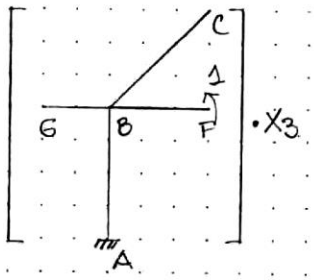
$-M_A - 1 \times 4 = 0$

$M_A = -4$

DIAGRAMA DE MOMENTOS



-Sistema Tres



$\sum M_A^{\circlearrowleft} = 0$

$-M_A + 1 = 0$

$M_A = 1$

$\sum M_B^{\circlearrowleft} = 0 \quad -M_{BF} + 1 = 0$

$M_{BF} = 1$

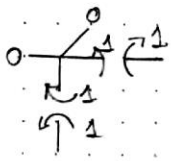
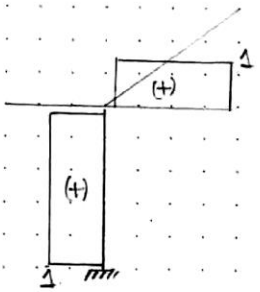


DIAGRAMA DE MOMENTOS



Un análisis rápido permite concluir que al no existir una reacción horizontal en "A", directamente el diagrama de momentos en AB es constante, puesto que no existe fuerza alguna capaz de generar una solicitación de momento flector adicional en el extremo B además del momento en "A".

El Teorema de Superposición indica que:

$$\begin{cases} D_1 = D_{10} + d_{11} \cdot X_1 + d_{12} \cdot X_2 + d_{13} X_3 \\ D_2 = D_{20} + d_{21} \cdot X_1 + d_{22} X_2 + d_{23} \cdot X_3 \\ D_3 = D_{30} + d_{31} \cdot X_1 + d_{32} X_2 + d_{33} \cdot X_3 \end{cases}$$

$$D_{10} = \frac{(2\times 4000\times(-7) + 2\times 4000\times(-3) + (-7)\times 4000 + (-3)\times 4000)\times 4}{6EI_0} = -\frac{80.000}{EI_0}$$

$$d_{11} = \frac{(-3)\times(-3)\times 5}{3EI_0} + \frac{[2\times(-3)\times(-3) + 2\times(-7)\times(-7) + (-3)\times(-7) + (-7)\times(-3)]\times 4}{6EI_0} = \frac{361}{3EI_0}$$

$$d_{12} = \frac{(2\times(-4)\times(-7) + 2\times(0)\times(-3) + (-4)\times(-3) + (0)\times(-7))\times 4}{6EI_0} = \frac{136}{3EI_0}$$

$$d_{13} = \frac{(2\times(-7)\times 1 + 2\times(-3)\times 1 + (-7)\times 1 + (-3)\times 1)\times 4}{6EI_0} = -\frac{20}{EI_0}$$

PREPARADOR: REDUCCIÓN DE VIGA R

$$D_{20} = \frac{(-4)\times 4000\times 4}{2EI_0} = -\frac{32000}{EI_0}$$

$$d_{22} = \frac{(-4)\times(-4)\times 4}{3EI_0} = \frac{64}{3EI_0}$$

$$d_{23} = \frac{(-4)\times 1\times 4}{2EI_0} = -\frac{8}{EI_0}$$

$$D_{30} = \frac{1\times 4000\times 4}{EI_0} = \frac{16000}{EI_0}$$

$$d_{33} = \frac{1\times 1\times 4}{EI_0} + \frac{1\times 1\times 4}{EI_0} = \frac{8}{EI_0}$$

$$\begin{cases} 0 = -\frac{80.000}{EI_0} + \frac{36}{3EI_0}X_1 + \frac{136}{3EI_0}X_2 - \frac{20}{EI_0}X_3 & (I) \\ 0 = -\frac{32000}{EI_0} + \frac{136}{3EI_0}X_1 + \frac{64}{3EI_0}X_2 - \frac{8}{EI_0}X_3 & (II) \\ 0 = \frac{16000}{EI_0} - \frac{20}{EI_0}X_1 - \frac{8}{EI_0}X_2 + \frac{8}{EI_0}X_3 & (III) \end{cases}$$

$$X_1 = 432 \text{ Kg}$$

$$X_2 = 378 \text{ Kg}$$

$$X_3 = -541 \text{ Kg-m}$$

$$M_A = M_A^{(0)} + M_A^{(1)}\cdot X_1 + M_A^{(2)}\cdot X_2 + M_A^{(3)}\cdot X_3 = -1077 \text{ Kg-m}$$

$$M_{BC} = M_{BC}^{(0)} + M_{BC}^{(1)}\cdot X_1 + M_{BC}^{(2)}\cdot X_2 + M_{BC}^{(3)}\cdot X_3 = -1296 \text{ Kg-m}$$

$$M_{BG} = M_{BG}^{(0)} + M_{BG}^{(1)}\cdot X_1 + M_{BG}^{(2)}\cdot X_2 + M_{BG}^{(3)}\cdot X_3 = -4000 \text{ Kg-m}$$

$$M_{BF} = M_{BF}^{(0)} + M_{BF}^{(1)}\cdot X_1 + M_{BF}^{(2)}\cdot X_2 + M_{BF}^{(3)}\cdot X_3 = -541 \text{ Kg-m}$$

$$M_{BA} = M_{BA}^{(0)} + M_{BA}^{(1)}\cdot X_1 + M_{BA}^{(2)}\cdot X_2 + M_{BA}^{(3)}\cdot X_3 = 2163 \text{ Kg-m}$$

$$M_{FB} = M_{FB}^{(0)} + M_{FB}^{(1)}\cdot X_1 + M_{FB}^{(2)}\cdot X_2 + M_{FB}^{(3)}\cdot X_3 = -541 \text{ Kg-m}$$

PREPARADO: REDECAL J. VIREL R

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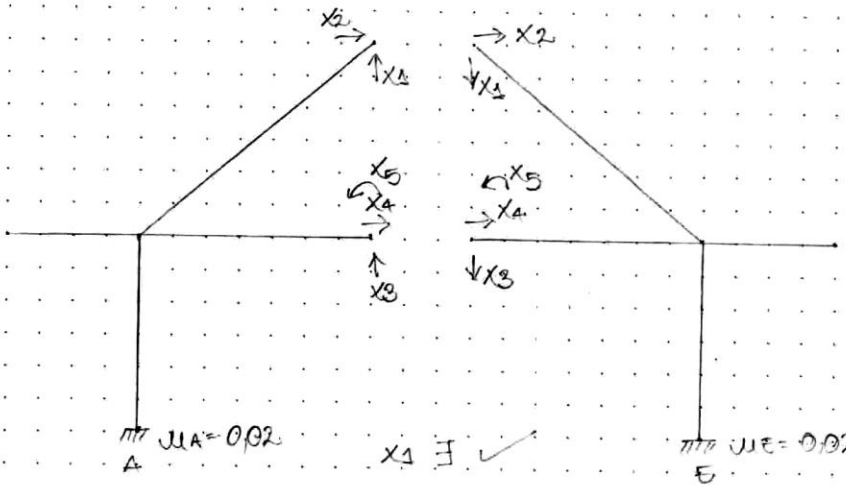
• ESTRUCTURA SIMÉTRICA CON CARGAS ANTIMÉTRICAS

- Se plantea el corte de la estructura



$X_1 \exists$ ✓

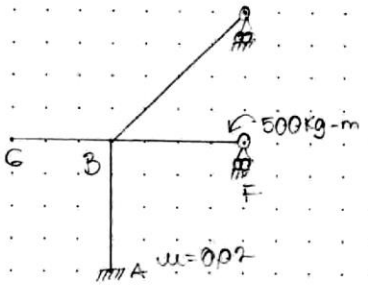
$X_2 = 0$

$X_3 \exists$ ✓

$X_4 = 0$

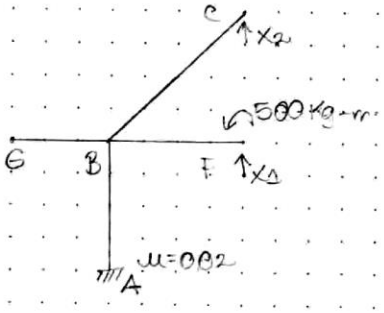
$2X_5 = 1000\ kg\cdot m$

- El sistema equivale a:



• MÉTODO DE LAS FUERZAS

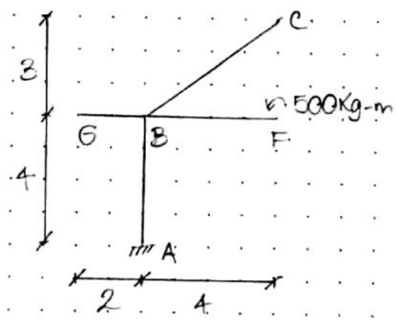
- Sistema Primario



Ec de compatibilidad:

El trabajo realizado por las fuerzas X_1 y X_2 es cero. $D_1 = D_2 = 0$

- Sistema Cero.



$\sum M_{B_{BF}} = 0 \Rightarrow -M_{BF} + 500 = 0$

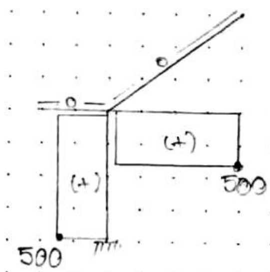
$M_{BF} = 500 \text{Kg-m}$

$\sum M_A = 0 \Rightarrow -M_A + 500 = 0$

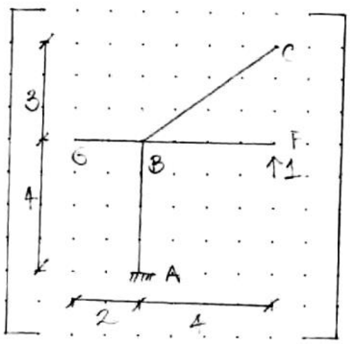
$M_A = 500$

$\sum F_h = 0 \Rightarrow A_h = 0 \Rightarrow M_{BA} = M_A$

DIAGRAMA DE MOMENTOS



- Sistema Uno.



• X1

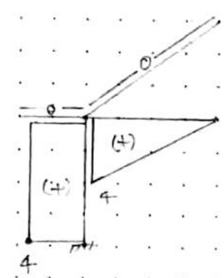
$\sum M_{B_{BF}} = 0 \Rightarrow -M_{BF} + 1 \times 4 = 0$

$M_{BF} = 4$

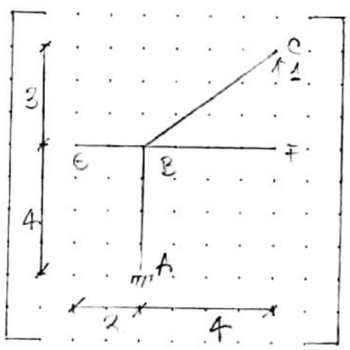
$\sum M_A = 0 \Rightarrow -M_A + 1 \times 4 = 0$

$M_A = 4$

DIAGRAMA DE MOMENTOS



- Sistema Dos.



• X2

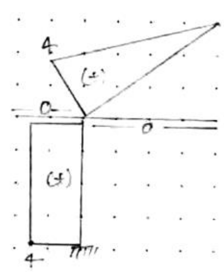
$\sum M_{B_{BC}} = 0 \Rightarrow -M_{BC} + 1 \times 4 = 0$

$M_{BC} = 4$

$\sum M_A = 0 \Rightarrow -M_A + 1 \times 4 = 0$

$M_A = 4$

DIAGRAMA DE MOMENTOS



El Teorema de Superposición indica que

$$\begin{cases} D_1 = D_{10} + d_{11} X_1 + d_{12} X_2 \\ D_2 = D_{20} + d_{21} X_1 + d_{22} X_2 \end{cases}$$

$$D_{10} = \frac{500 \times 4 \times 4}{EI_0} + \frac{500 \times 4 \times 4}{2EI_0} = 12000/EI_0$$

$$d_{11} = \frac{4 \times 4 \times 4}{EI_0} + \frac{4 \times 4 \times 4}{3EI_0} = \frac{256}{3EI_0}$$

$$d_{12} = \frac{4 \times 4 \times 4}{EI_0} = 64/EI_0$$

$$D_{20} = \frac{500 \times 4 \times 4}{EI_0} = 8000/EI_0$$

$$d_{22} = \frac{4 \times 4 \times 4}{EI_0} + \frac{4 \times 4 \times 5}{3EI_0} = \frac{272}{3EI_0}$$

$$\begin{cases} 0 = \frac{12000}{EI_0} + \frac{256}{3EI_0} \cdot X_1 + \frac{64}{EI_0} \cdot X_2 \\ 0 = \frac{8000}{EI_0} + \frac{64}{EI_0} \cdot X_1 + \frac{272}{3EI_0} X_2 \end{cases}$$

$X_1 = -158\ Kg$

$X_2 = 23\ Kg$

$M_{BC} = 4X_2 = 92\ Kg\text{-}m$

$M_{BF} = 500 + 4X_1 = -132\ Kg\text{-}m$

$M_{BA} = 500 + 4X_1 + 4X_2 = -40\ Kg\text{-}m$

$M_{AB} = 500 + 4X_1 + 4X_2 = -40\ Kg\text{-}m$

$M_{FB} = 500\ Kg\text{-}m$

PREPARADOR REFORZAR Y/REC 2

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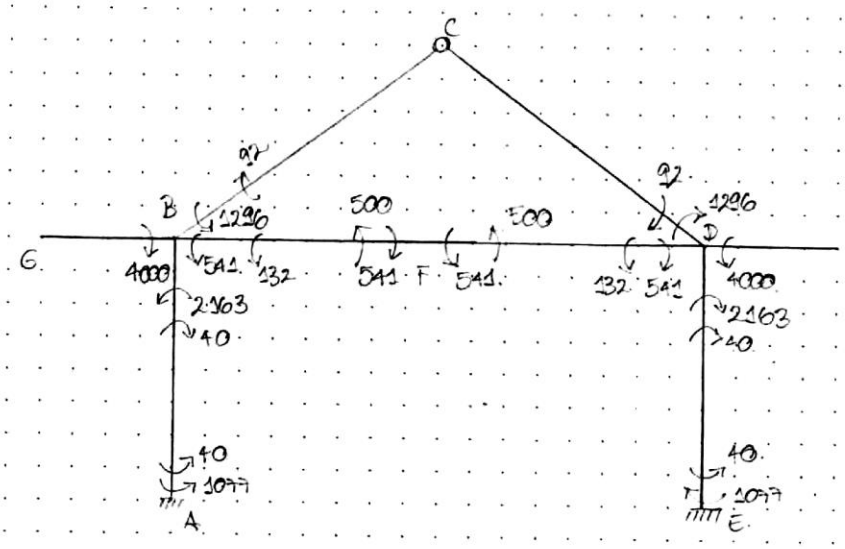
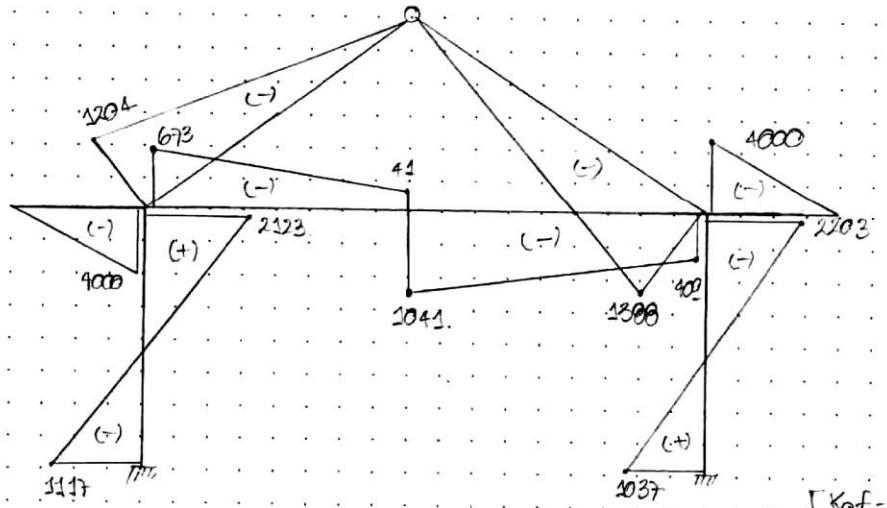


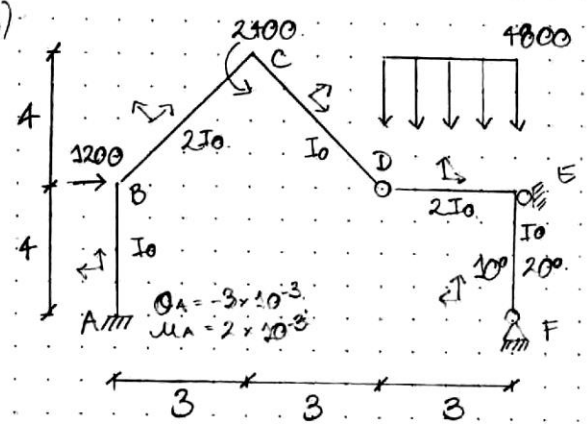
DIAGRAMAS FINALES.



[Kgf-m]

PREPARADO POR REBECA J. VIPEL R.

3)



Unidades:

Fuerza [Kg]

Longitud [m]

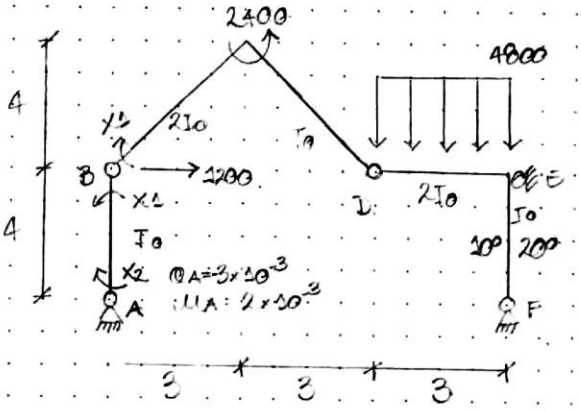
Temperatura [°C]

$EI_o = 1\times10^6\ Kg-m^2$

Por inspección se tiene que: $GH = 2$

MÉTODO DE LAS FUERZAS.

-Sistema Primario



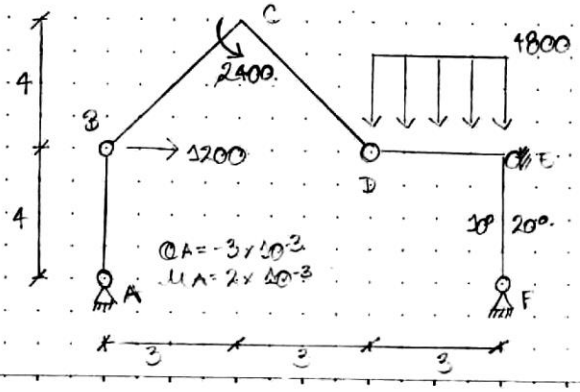
Ecuaciones de compatibilidad

$D_1 = 0$

$D_2 = 3\times10^{-3}$

El trabajo realizado por la fuerza X1 es cero.

-Sistema Cero



Por inspección $Ah = 0$

$\sum M_D^{dt} = 0 \Rightarrow -Av\times6 + Ah\times4 + 2400 = 0$

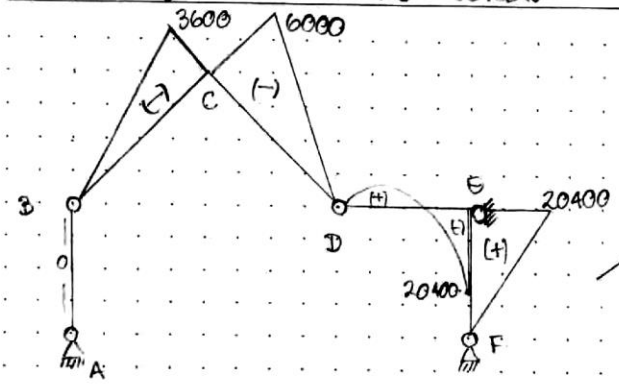
$Av = 400\ Kg$

$\sum M_C^{dt} = 0 \Rightarrow M_{CB} + 1200\times4 + Ah\times8 - Av\times3 = 0$

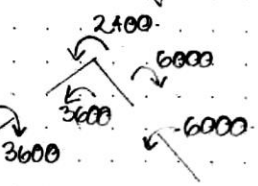
$M_{CB} = -3600\ Kg-m$

$\sum M_E^{dt} = 0: \ 4800\times3\times\frac{3}{2} + 2400 + Ah\times4 - Av\times9 + M_{DE} = 0$

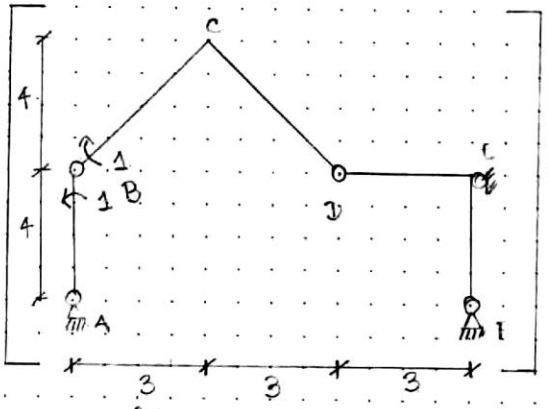
$M_{DE} = -20400\ Kg-m$



Equilibrio junta C



- Sistema Uno



$\cdot X_1$

$\overset{\curvearrowright}{\sum M_B} = 0 \quad 1 + Ah \times 4 = 0 \Rightarrow Ah = -1/4$

$\overset{\curvearrowright}{\sum M_D} = 0 \quad Ah \times 4 - Av \times 6 = 0 \Rightarrow Av = -1/6$

$\overset{\curvearrowright}{\sum M_E} = 0 \quad M_{CB} + Ah \times 8 - Av \times 3 = 0$

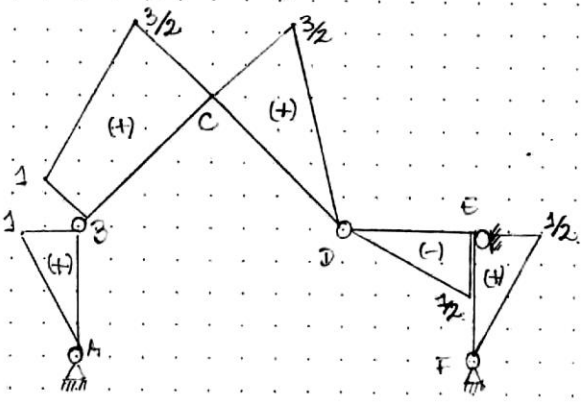
$M_{CB} = 3/2$

$\overset{\curvearrowright}{\sum M_C} = 0 \quad -Av \times 3 + Ah \times 4 + M_{CD} = 0$

$M_{CD} = -1/2$

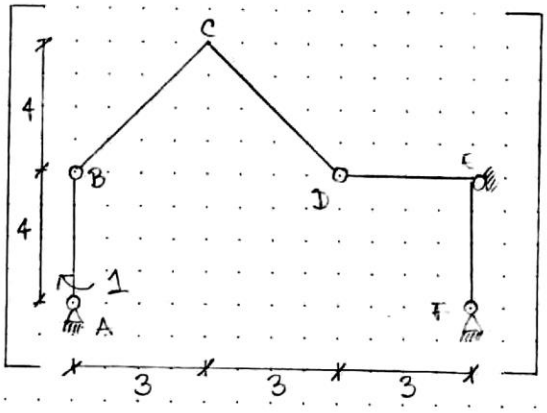
$+\uparrow \sum F_V = 0$

$Av + Fv = 0 \Rightarrow Fv = 1/6$



PREPARADOR: REDUCIR EL NIVEL R

-Sistema Dos.



$\curvearrowright \sum M_B = 0 \quad -1 + Ah \times 4 = 0 \Rightarrow Ah = 1/4$

$\sum M_D = 0 \quad -1 + Ah \times 4 - Av \times 6 = 0 \Rightarrow Av = 0$

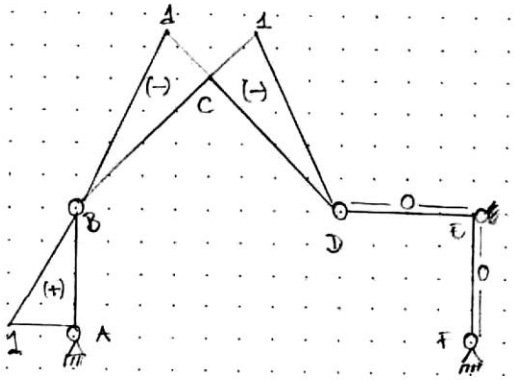
$\cdot X_2 \quad \sum M_C = 0 \quad M_{CB} - Av \times 3 + Ah \times 2 - 1 = 0$

$M_{CB} = -1$

$\sum M_E = 0 \quad M_{ED} - 1 + Ah \times 4 - Av \times 9 = 0$

$M_{ED} = 0$

$+\uparrow \sum F_V = 0 \quad Av + Fv = 0 \Rightarrow Fv = 0$



Aplicando el Teorema de superposición se tiene que:

$$\begin{cases} D_1 = D_{10} + d_{11} X_1 + d_{12} X_2 \\ D_2 = D_{20} + d_{21} X_1 + d_{22} X_2 \end{cases}$$

$$D_{10} - \frac{1}{4} \times 2 \times 10^{-3} \times \frac{10^6}{EI_0} = \frac{5}{6 \times 2EI_0}\left[2 \times (3/2) \times (-3600) + (-3600) \times (1)\right] + \frac{(-6000) \times (3/2) \times 5}{3EI_0} +$$

$$\frac{(-1/2) \times (-20400) \times 3}{3 \times 2EI_0} + \frac{(-1/2) \times (5400) \times 3}{3 \times 2EI_0} + \frac{20400 \times 1/2 \times 4}{3EI_0} +$$

$$\left[10^{-5} \times 15 \times (-1/6 \times 4) + \frac{10^{-5} \times 10}{0{,}5} \times \frac{1/2 \times 4}{2}\right] \times \frac{10^6}{EI_0}$$

$D_{10} = -3050/EI_0$

PREPARADOR: REDESAR ... V/REL R

$$\delta_{11} = \frac{1\times1\times4}{3EI_0} + \frac{5}{6\times2EI_0}\left[2\times1\times1 + 2\times\frac{3}{2}\times\frac{3}{2} + 2\times1\times\frac{3}{2}\right] + \frac{\frac{3}{2}\times\frac{3}{2}\times5}{3EI_0} + \frac{(-\frac{1}{2})\times(-\frac{1}{2})\times3}{2\times2EI_0}$$

$$+ \frac{\frac{1}{2}\times\frac{1}{2}\times4}{3EI_0}$$

$$\delta_{11} = \frac{19}{2EI_0}$$

$$\delta_{12} = \frac{1\times1\times4}{6EI_0} + \frac{5}{6\times2EI_0}\left[2\times(\tfrac{3}{2})\times(-1) + (-1)\times(1)\right] + \frac{\frac{3}{2}\times(-1)\times5}{3\times EI_0}$$

$$\delta_{12} = \frac{-7}{2EI_0}$$

$$D_{20} + \frac{1}{4}\times2\times10^{3}\times10^{6} = \frac{(-3600)\times(-1)\times5}{3\times2EI_0} + \frac{(-6000)\times(-1)\times5}{3EI_0}$$

$$D_{20} = 12500/EI_0$$

$$\delta_{22} = \frac{1\times1\times4}{3EI_0} + \frac{(-1)\times(-1)\times5}{3\times2EI_0} + \frac{(-1)\times(-1)\times5}{3EI_0}$$

$$\delta_{22} = \frac{23}{6EI_0}$$

$$\begin{cases} 0 = -\frac{3050}{EI_0} + \frac{19}{2EI_0}X_1 - \frac{7}{2EI_0}X_2 \quad (I) \\ \frac{3000}{EI_0} = \frac{12500}{EI_0} - \frac{7}{2EI_0}X_1 + \frac{23}{6EI_0}X_2 \quad (II) \end{cases}$$

$$X_1 = -892\ Kg\text{-}m \;;\quad X_2 = -3293\ Kg\text{-}m$$

Aplicando el teorema de superposición se tiene que:

$$M_{CB} = M_{CB}^{(0)} + M_{CB}^{(1)}X_1 + M_{CB}^{(2)}X_2$$

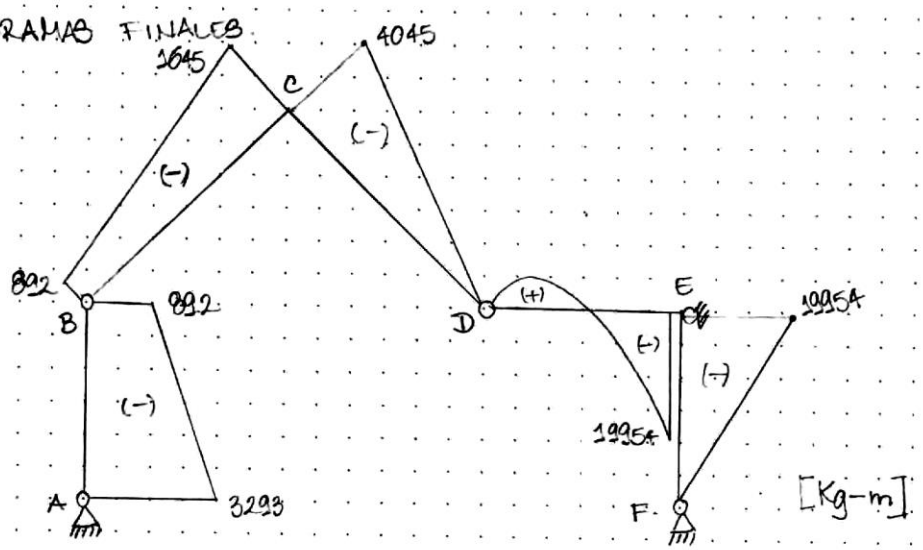
$$M_{CB} = -3000 + \tfrac{3}{2}\times(-892) - 1\times(-3293) \Rightarrow M_{CB} = -1645\ Kg\text{-}m$$

$$M_{ED} = M_{ED}^{(0)} + M_{ED}^{(1)}X_1 + M_{ED}^{(2)}X_2$$

$$M_{ED} = -20400 - \frac{1}{2}\times(-892) \Rightarrow M_{ED} = -19954\ Kg\text{-}m$$

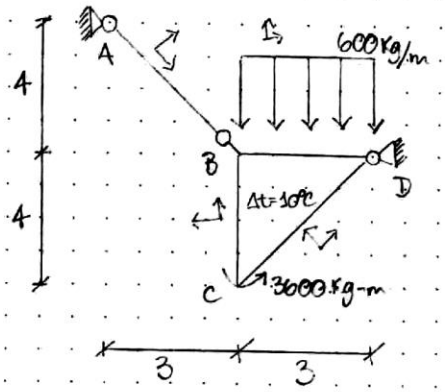
PREPARADOR: REDESCAR J. VIRELA R.

o DIAGRAMAS FINALES



PREPARADO: REDECLAR J. VIRELLA

4) Determine los diagramas de Momento.



$h = \frac{1}{2}$ m

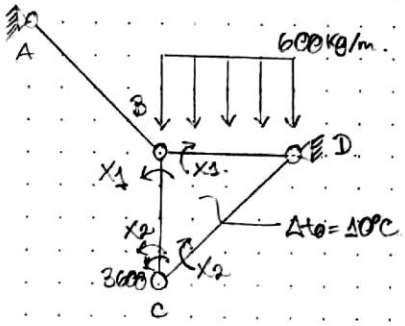
$\alpha t = 1 \times 10^{-6}\ °c^{-1}$

$\mu_A = 2 \times 10^{-2}$ m

MÉTODO DE LAS FUERZAS

-Por inspección se obtiene que la estructura posee 2 grados de hiperestaticidad. El sistema aislado ABD conforma un sistema isostático, al incorporar el conjunto BCD se establece una hiperestaticidad que puede ser entendida como la restricción adicional en el extremo D del miembro BD, de sus componentes de desplazamiento traslacional ($\leftrightarrow$ y $\updownarrow$); por ello se tienen dos grados de hiperestaticidad

-Sistema Primario.



$\mu_A = 2 \times 10^{-2}$

Ec. de compatibilidad

$D1 = 0$

$D2 = 0$

El trabajo producido por las fuerzas X_1 y X_2 es nulo

PREPARADOR: REDESCAR F. VIREL R

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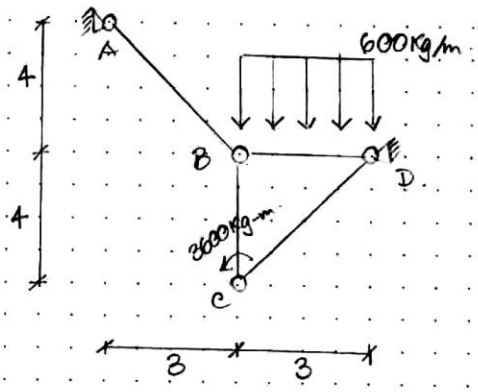
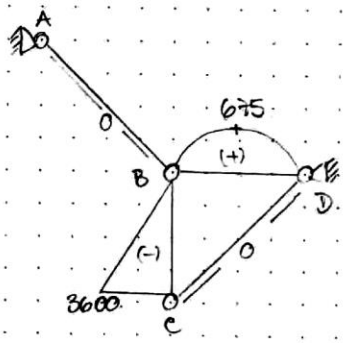
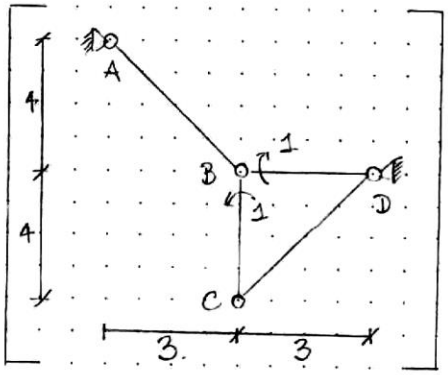
- Sistema Cero

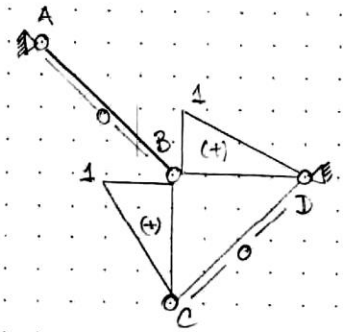




- Sistema Uno



$\cdot X_1$



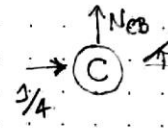
Puesto que como dato se aprecia un cambio de temperatura en el conjunto ECV, se deben hallar las fuerzas axiales de cada barra, así como la reacción horizontal en A asociada al movimiento de apoyo en los sistemas 1 y 2

$$\begin{cases} \sum M_D^{\circlearrowleft} = 0 \Rightarrow -Ah \times 4 - Av \times 6 = 0 \quad (I) \\ \sum M_B^{\circlearrowleft} = 0 \Rightarrow -Ah \times 4 - Av \times 3 = 0 \quad (II) \end{cases}$$

$Ah = 0 \; ; \; Av = 0$

PREPARADOR: REDUCCIÓN I VIGA 2

NODO "C"



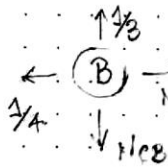
$M_{3(x)}^{CB} = \frac{1}{4}x \Rightarrow y_{CB}^{(x)} = \frac{1}{4}$ $\quad \sin\alpha = 4/5 \; ; \cos\alpha = 3/5$

$\overset{+}{\rightarrow}\sum Fh = 0 \quad V_{CB} + N_{CD}\cos\alpha = 0$

$\frac{1}{4} + N_{CD} \times 3/5 = 0 \Rightarrow N_{CD} = -5/12$

$+\uparrow\sum Fv = 0 \quad N_{CB} + N_{CD}\sin\alpha = 0 \Rightarrow N_{CB} = 1/3$

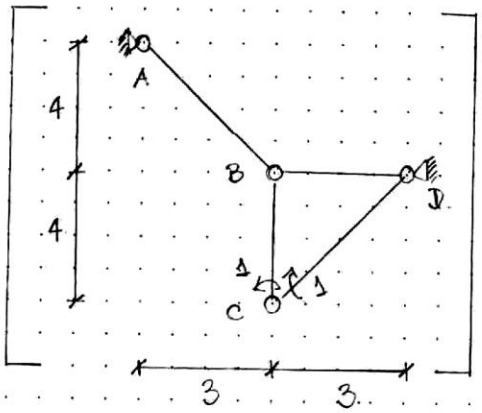
NODO "B"



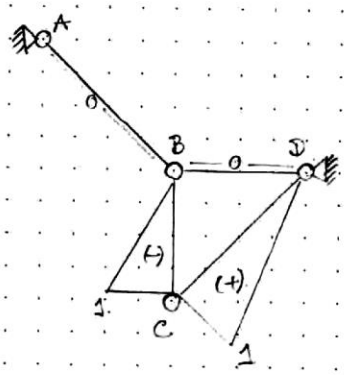
$M_{3(x)}^{BD} = -\frac{1}{3}x + 1 \Rightarrow y_{BD}^{(x)} = -1/3$

$\overset{+}{\rightarrow}\sum Fh = 0 \quad N_{BD} - 1/4 = 0 \Rightarrow N_{BD} = 1/4$

- Sistema Dos

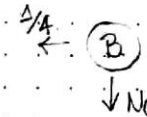


$\times 2$



Por inspección se tiene que $A_h = 0$

NODO "B"



$M_{3(x)}^{CB} = \frac{1}{4}x - 1 \Rightarrow y_{CB}^{(x)} = 1/4$

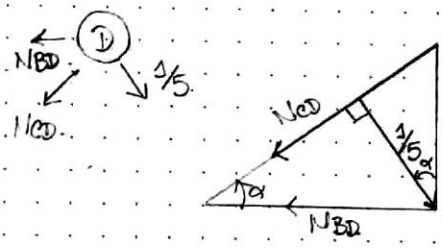
$\overset{+}{\rightarrow}\sum Fh = 0 \Rightarrow N_{BD} - 1/4 = 0 \Rightarrow N_{BD} = 1/4$

$+\uparrow\sum Fv = 0 \Rightarrow N_{CB} = 0$

PREPARADOR: REVISADO 1 V/REL 2

MODO "D"

$$M_{S_{CD}}(x) = -\frac{1}{5}x + 1 \Rightarrow y_{CD}(x) = -\frac{1}{5}$$



$\sin\alpha = 4/5$; $\cos\alpha = 3/5$

$$+\uparrow \sum F_V = 0.$$

$$-N_{CD} \times \sin\alpha - \frac{1}{5} \times \cos\alpha = 0$$

$$N_{CD} = -\frac{3}{20}$$

El teorema de superposición exigiere que:

$$\begin{cases} D_1 = D_{10} + d_{11} \cdot X_1 + d_{12} \cdot X_2 \\ D_2 = D_{20} + d_{21} \cdot X_1 + d_{22} \cdot X_2 \end{cases}$$

$$D_{10} = \frac{1 \times (675) \times 3}{3EI_0} + \frac{1 \times (-3600) \times 4}{6EI_0} + 10^{-6} \times 5 \times \frac{10^6}{EI_0} \times \left[\frac{1}{4} \times 3 + \left(-\frac{5}{12} \times 5\right) + \frac{1}{3} \times 4\right] + \ldots$$

$$\ldots + \frac{10^{-6}}{0{,}5} \times \left[(+10) \times \frac{1 \times 3}{2} + (10) \times \frac{1 \times 4}{2}\right] \times \frac{10^6}{EI_0}$$

$$D_{10} = -1655/EI_0$$

$$d_{11} = \frac{1 \times 1 \times 3}{3EI_0} + \frac{1 \times 1 \times 4}{3EI_0} \Rightarrow d_{11} = 7/3EI_0$$

$$d_{12} = \frac{1 \times (-1) \times 4}{6EI_0} \Rightarrow d_{12} = -2/3EI_0$$

$$D_{20} = \frac{(-1) \times (-3600) \times 4}{3EI_0} + 10^{-6} \times 5 \times \frac{10^6}{EI_0} \left[\frac{1}{4} \times 3 + \left(-\frac{3}{20}\right) \times 5\right] + \ldots$$

$$\ldots + \frac{10^{-6}}{0{,}5} \times \frac{10^6}{EI_0} \times \left[\frac{10 \times (-1) \times 4}{2} + (-10) \times \frac{(1 \times 5)}{2}\right]$$

$$D_{20} = 4710/EI_0$$

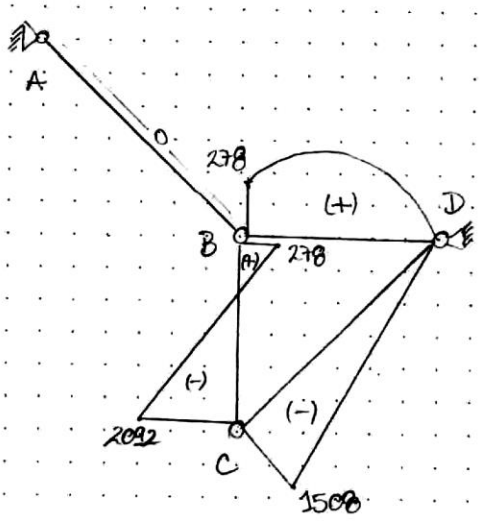
PREPARADOR: REPASO 3 VIGA 2

$$\delta_{22} = \frac{(-1)\times(-1)\times 4}{3EI_0} + \frac{1\times 1\times 5}{3EI_0} \Rightarrow \delta_{22} = \frac{3}{EI_0}$$

$$\begin{cases} 0 = -\frac{1655}{EI_0} + \frac{7}{3EI_0} X_1 - \frac{2}{3EI_0} \cdot X_2 \quad (I) \\ 0 = \frac{4710}{EI_0} - \frac{2}{3EI_0} X_1 + \frac{3}{EI_0} \cdot X_2 \quad (II) \end{cases}$$

$$X_1 = 278\,Kg\text{-}m \;;\; X_2 = -1508\,Kg\text{-}m$$

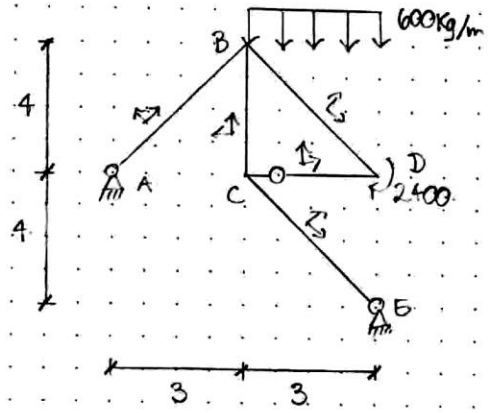
- DIAGRAMAS FINALES.

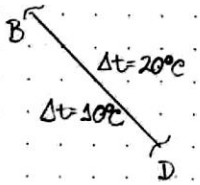


[Kg-m]

PREPARADO: REDESAR S. V/REL R

5). Determine el diagrama de Momentos Flectores





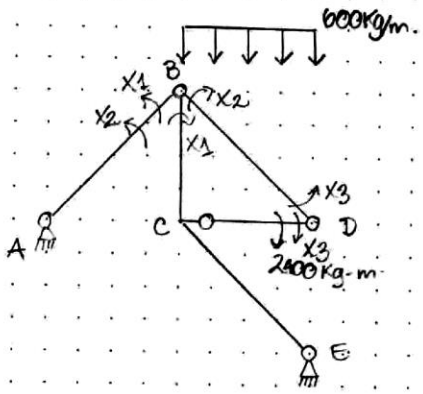
$\alpha t = 1 \times 10^{-5} °C^{-1}$

$h = 0{,}60\ m$

- MÉTODO DE LAS FUERZAS

Por inspección se identifica sistema estructural de 3 grados de hiperestaticidad.

- Sistema Primario.



Ecuaciones de Compatibilidad

$D_1 = D_2 = D_3 = 0$

El trabajo producido por las fuerzas internas X_1, X_2 y X_3 es cero.

PREPARADOR: REDESCAR S. VIZCAL R

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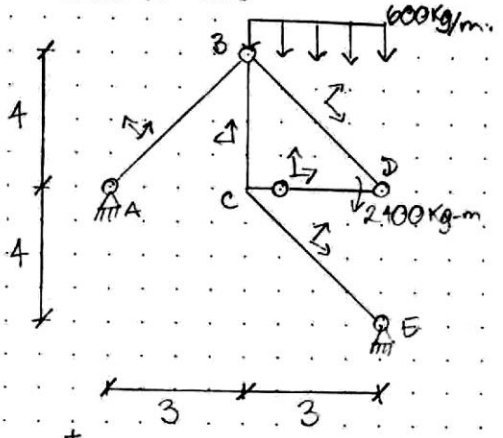
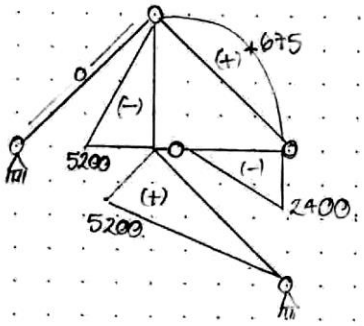
- Sistema Cero





$\sum M_A = 0 \Rightarrow Eh \times 4 + Ev \times 6 - 2400 - 600 \times 3 \times 4{,}5 = 0 \Rightarrow 4Eh + 6Ev = 10500$ (I)

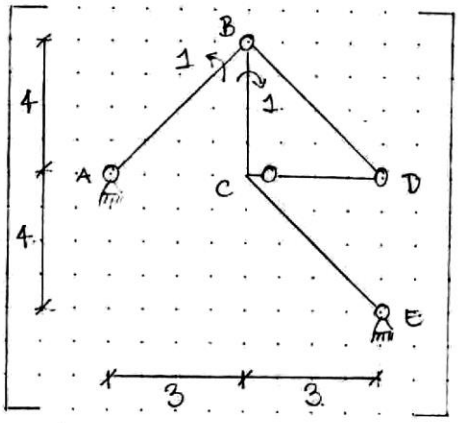
$\sum M_B = 0 \Rightarrow -2400 - 600 \times 3 \times \frac{3}{2} + Eh \times 8 + Ev \times 3 = 0 \Rightarrow 8Eh + 3Ev = 5100$ (II)

$Eh = -25\,Kg \quad Ev = 5800/3\,Kg.$

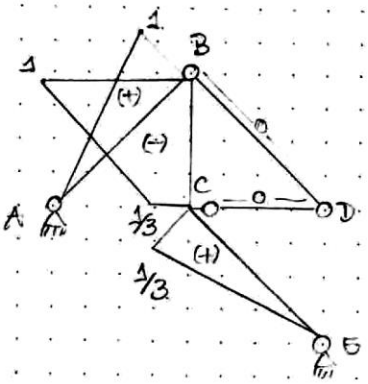
$\sum M_{C_{CE}} = 0 \quad -M_{CE} + Ev \times 3 + Eh \times 4 = 0$

$M_{CE} = 5200\,Kg\text{-}m$

- Sistema Uno.



$\cdot X1$

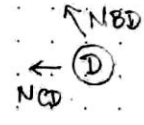


$\sum M_A = 0 \quad Eh \times 4 + Ev \times 6 = 0 \quad (I) \qquad Eh = 1/6$

$\sum M_B = 0 \quad -1 + Eh \times 8 + Ev \times 3 = 0 \quad (II) \qquad Ev = -1/9$

$\sum M_{C_{CE}} = 0 \quad -M_{CE} + Eh \times 4 + Ev \times 3 = 0 \Rightarrow M_{CE} = 1/3$

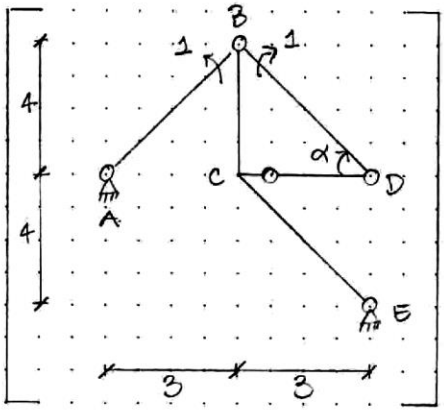
NODO "D"



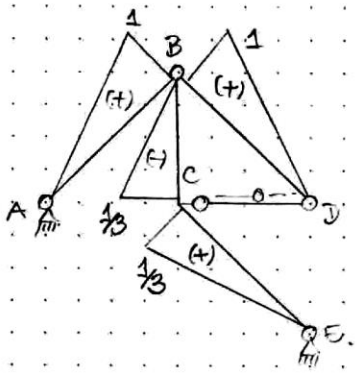
Por inspección $N_{BD} = 0$

PREPARADOR: REDEDUCAR I. VIREL R.

- Sistema Dos.



$\cdot X_2$

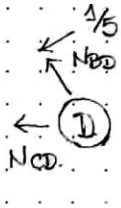


$\sum M_A^{\alpha\uparrow} = 0 \quad Eh \times 4 + Ev \times 6 = 0 \quad (I) \quad Eh = 1/6$

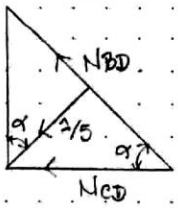
$\sum M_B^{\alpha\uparrow} = 0 \quad -1 + Eh \times 8 + Ev \times 3 = 0 \quad (II) \quad Ev = -1/9$

$\sum M_{C_{CE}}^{\alpha\curvearrowright} = 0 \quad -M_{CE} + Eh \times 4 + Ev \times 3 = 0 \Rightarrow M_{CE} = 1/3$

NODO "D"



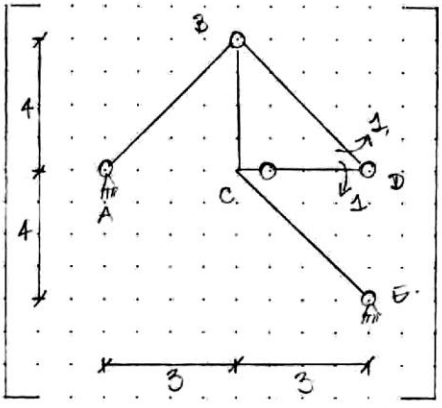
$M_{3(x)}_{BD} = -1/5x + 1 \Rightarrow V_{y(x)}_{BD} = -1/5 \quad \text{sen}\alpha = 4/5 \; ; \; \cos\alpha = 3/5$



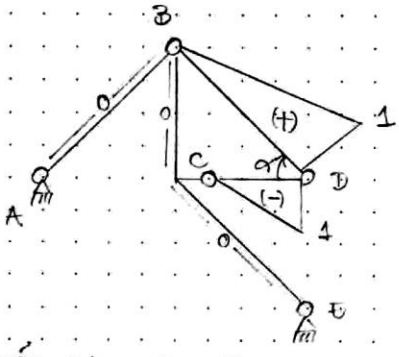
$+\uparrow \sum Fv = 0 \quad NBD \times \text{sen}\alpha - 1/5 \times \cos\alpha = 0$

$NBD = 3/20.$

- Sistema Tres



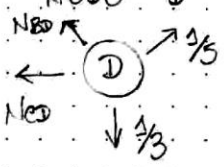
$\cdot X_3$



Por inspección Eh y $Ev = 0$.

PREPARADOR: R. EDGAR J. VIRET R.

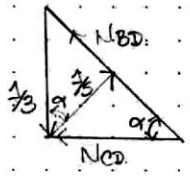
NODO "D"



$\mu_{3(x)}_{CD} = -1/3\,x \Rightarrow y_{(x)}_{CD} = -1/3$

$\mu_{3(x)}_{BD} = 1/5\,x \Rightarrow y_{(x)}_{BD} = 1/5$

$\text{sen}\alpha = 4/5 \quad \cos\alpha = 3/5$



$+\uparrow \sum F_V = 0 \quad N_{BD}\times\text{sen}\alpha - 1/3 + 4/5 \times \cos\alpha = 0 \Rightarrow N_{BD} = 4/15.$

$$\begin{cases} D_1 = D_{10} + d_{11}X_1 + d_{12}X_2 + d_{13}X_3 \\ D_2 = D_{20} + d_{21}X_1 + d_{22}X_2 + d_{23}X_3 \\ D_3 = D_{30} + d_{31}X_1 + d_{32}X_2 + d_{33}X_3 \end{cases}$$

$$D_{10} = \frac{1/3 \times 5200 \times 5}{3EI_0} + \frac{4}{6}\times\left[2\times(-1/3)\times(-5200) + (-1)\times(-5200)\right]\times\frac{1}{EI_0}$$

$$D_{10} = 26000/3EI_0$$

$$d_{11} = \frac{1\times1\times5}{3EI_0} + \frac{1}{3}\times\frac{1}{3}\times\frac{5}{3EI_0} + \frac{4}{6}\left[2\times(-1/3)\times(-1/3) + 2\times(-1)\times(-1) + 2\times(-1)\times(-1/3)\right]$$

$$d_{11} = 34/9\,EI_0$$

$$d_{12} = \frac{1\times1\times5}{3EI_0} + \frac{1}{3}\times\frac{1}{3}\times\frac{5}{3EI_0} + \frac{4}{6}\left[2\times(-1/3)\times(-1/3) + (-1)\times(-1/3)\right]$$

$$d_{12} = 10/9\,EI_0$$

$$d_{13} = 0$$

$$D_{20} = \frac{(-1/3)\times(-5200)\times4}{3EI_0} + \frac{1\times675\times5}{3EI_0} + \frac{5200\times1/3\times5}{3EI_0} + \frac{10^{-5}\times10^{6}}{EI_0}\times\left[3/20\times5\right]\times15$$

$$+\frac{10^{-5}\times10^{6}}{0,60\times EI_0}\times\left[\frac{1\times5}{2}\right]\times(-10)$$

$$D_{20} = 36125/6EI_0$$

PREPARADO: REDUCCIÓN = 1/REL X

$$d_{22} = \frac{1\times1\times5}{3EI_0} + \frac{\frac{1}{3}\times\frac{1}{3}\times5}{3EI_0} + \frac{(-\frac{1}{3})\times(-\frac{1}{3})\times4}{3EI_0}$$

$$d_{22} = 11/3EI_0$$

$$d_{23} = \frac{1\times1\times5}{6EI_0} = \frac{5}{6EI_0}$$

$$D_{30} = \frac{1\times(675\times5}{3EI_0} + \frac{(-2400)\times(-1)\times3}{3EI_0} + \frac{10^{-5}\times10^{6}}{EI_0}\times\frac{4}{15}\times5\times(15) + \frac{10^{-5}\times10^{6}}{0,60EI_0}\times\frac{1\times5}{2}\times(-10)$$

$$D_{30} = 9925/3EI_0$$

$$d_{33} = \frac{1\times1\times5}{3EI_0} + \frac{(-1)\times(-1)\times3}{3EI_0}$$

$$d_{33} = 8/3EI_0$$

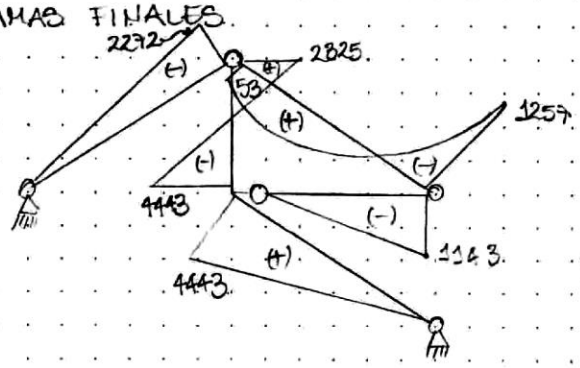
$$\begin{cases} 0 = 26000/3EI_0 + (34/3EI_0)X_1 + (20/3EI_0)X_2 & (I) \\ 0 = 36125/6EI_0 + (20/3EI_0)X_1 + (11/3EI_0)X_2 + (5/6EI_0)X_3 & (II) \\ 0 = 9925/3EI_0 + (5/6EI_0)X_2 + (8/3EI_0)X_3 & (III) \end{cases}$$

$X_1 = -2325\ Kg\text{-}m$; $X_2 = 53\ Kg\text{-}m$; $X_3 = -1257\ Kg\text{-}m$

$$M_{CE} = M_{CE}^{(0)} + M_{CE}^{(1)}\cdot X_1 + M_{CE}^{(2)}\cdot X_2 + M_{CE}^{(3)}\cdot X_3$$

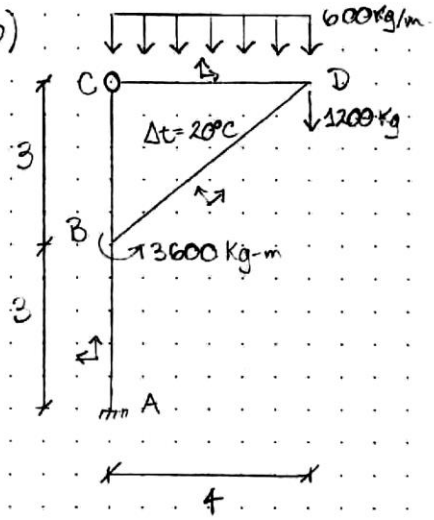
$$M_{CE} = 5200 + \frac{1}{3}\times(-2325) + \frac{1}{3}\times53 + (0)\cdot1257 \Rightarrow M_{CE} = 4443\ Kg\text{-}m\ (\curvearrowright)$$

○ DIAGRAMAS FINALES



PREPARADOR: REBESCAR T. V/REL 2.

6)



Determine los Diagramas de Momento

$\alpha t = 1 \times 10^{-5} °C^{-1}$

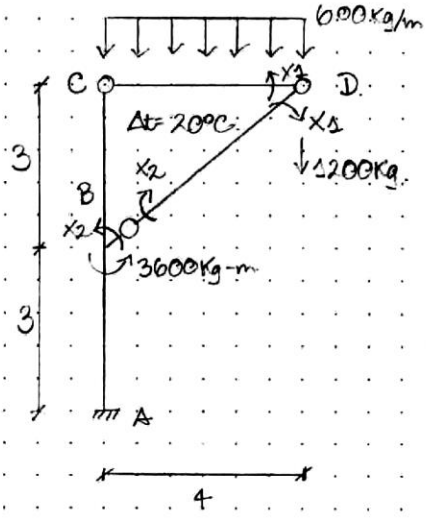
$h = 0,50m$

MÉTODO DE LAS FUERZAS:

$G_{LIBERTAD} = 3 \times$ nº barras $-$ (Vinternos + Vexternos).

$G_{LIBERTAD} = 3 \times 3 - (\underbrace{3}_{B} + \underbrace{3}_{D} + \underbrace{2}_{C} + \underbrace{3}_{A}) = -2 \Rightarrow$ 2 GRADOS DE HIPERESTATICIDAD.

-Sistema Primario



Ecuaciones de Compatibilidad.

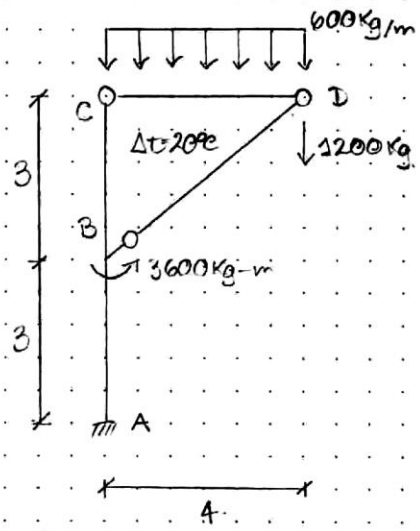
$D_1 = 0$

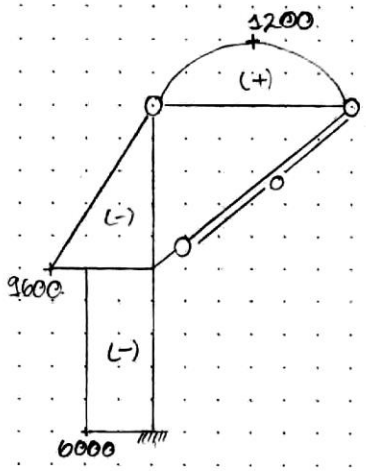
$D_2 = 0$

El trabajo realizado por las fuerzas X_1 y X_2 es cero

PREPARADOR: REDEFCOR J. YIBEL 2

-Sistema Cero

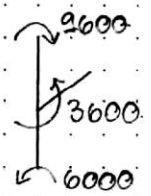




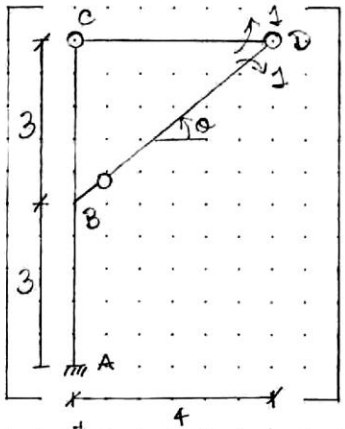
$\circlearrowleft \sum M_A = 0 \quad -M_A + 3600 - 1200 \times 4 - 600 \times 4 \times 2 = 0 \Rightarrow M_A = -6000$

Como $A_h = 0 \Rightarrow M_{BA} = -M_A$

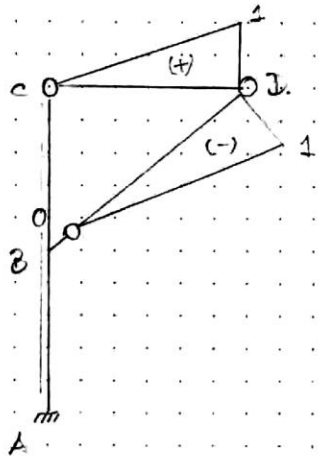
Equilibrio junta B



-Sistema Uno



$\cdot X_1$



$\circlearrowleft \sum M_A = 0 \Rightarrow M_A = 0$

PREPARADO: REVISAR I VIRIL R

Fuerzas Axiales de Interes.

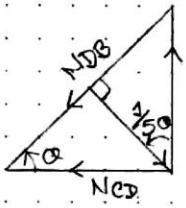
NODO "D"

$\leftarrow$ (D) $\uparrow 1/4$

N_{CD} $\swarrow$ N_{DB} $\searrow 1/5$

$M_{3(x)}^{CD} = \frac{1}{4}x \Rightarrow V_{y(x)}^{CD} = 1/4$

$M_{3(x)}^{DB} = -\frac{1}{5}x \Rightarrow V_{y(x)}^{DB} = -1/5$



$\sin\alpha = 3/5$

$\cos\alpha = 4/5$

$+\uparrow \sum F_V = 0 \quad -N_{DB}\times\sin\alpha - 1/5\times\cos\alpha + 1/4 = 0 \Rightarrow N_{DB} = 3/20$

$\overset{+}{\rightarrow} \sum F_h = 0 \quad 1/5\times\sin\alpha - N_{CD} - N_{DB}\times\cos\alpha = 0 \Rightarrow N_{CD} = 0$

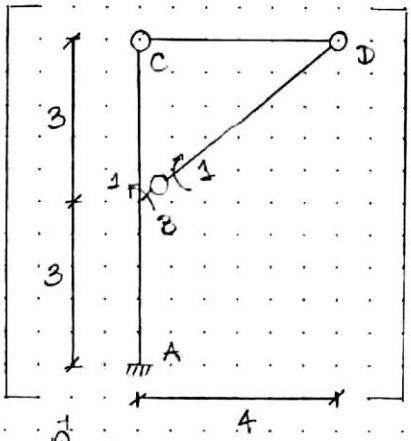
NODO "C"

$\downarrow 1/4$

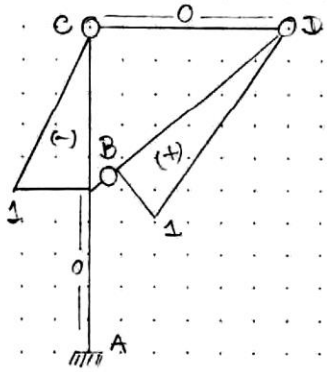
(C) $\rightarrow N_{CD}$ $+\uparrow \sum F_V = 0 \quad -1/4 - N_{CB} = 0 \Rightarrow N_{CB} = -1/4$

$\downarrow N_{CB}$

Sistema Dos.



$\cdot X_2$



$\overset{+}{\circlearrowleft} \sum M_A = 0 \Rightarrow M_A = 0 \Rightarrow M_{BA} = 0$ y $M_{BC} = 1$ (↺).

NODO "C"

$M_{3(x)}^{BC} = \frac{1}{3}x = 1 \Rightarrow V_{y(x)}^{BC} = 1/3$

$\leftarrow$ (C) $\rightarrow 1/3$

N_{CD}

$\downarrow N_{CB}$

$\overset{+}{\rightarrow} \sum F_h = 0 \Rightarrow N_{CD} = 1/3$

$+\uparrow \sum F_V = 0 \Rightarrow N_{CB} = 0$

PREPARADOR: REDEBAR T. V.REL 2

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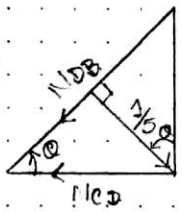
NODO "D"

$M_{3(x)}_{DB} = -1/5x + 1 \Rightarrow M_{y(x)}_{DB} = -1/5$



$\operatorname{sen}\theta = 3/5$

$\cos\theta = 4/5$

NCD ← (D) ↙ NDB, ↘ 1/5

$+\curvearrowleft \sum F_V = 0 \quad -N_{DB}\times\operatorname{sen}\theta - 1/5\times\cos\theta = 0 \Rightarrow N_{DB} = -4/15$

$$\begin{cases} D_1 = D_{10} + \delta_{11}X_1 + \delta_{12}X_2 \\ D_2 = D_{20} + \delta_{21}X_1 + \delta_{22}X_2 \end{cases} \leftarrow \text{PRINCIPIO DE SUPERPOSICIÓN}$$

$$D_{10} = \frac{1200\times1\times4}{3EI_\theta} + 10^{-5}\times10\times\frac{10^6}{EI_\theta}\times\left[\frac{-1}{4}\times3 + \frac{3}{20}\times5\right] + \frac{10^{-5}\times10^6}{0{,}5\times EI_\theta}\times\left[\frac{20\times1\times4}{2} - \frac{20\times(-1)\times5}{2}\right]$$

$$D_{10} = 3400/EI_\theta$$

$$\delta_{11} = \frac{1\times1\times4}{3EI_\theta} + \frac{(-1)\times(-1)\times5}{3EI_\theta}$$

$$\delta_{11} = \frac{8}{EI_\theta}$$

$$\delta_{12} = \frac{(-1)\times(1)\times5}{6EI_\theta} = \frac{-5}{6EI_\theta}$$

$$D_{20} = \frac{(-1)\times(-9600)\times3}{3EI_\theta} + 10^{-5}\times10\times\frac{10^6}{EI_\theta}\times\left[\frac{1}{8}\times4 - \frac{4}{15}\times5\right] + \frac{10^{-5}\times10^6}{0{,}5\times EI_\theta}\times\left[\frac{20\times(-1)\times3}{2} - \frac{20\times1\times5}{2}\right]$$

$$D_{20} = 8000/EI_\theta$$

$$\delta_{22} = \frac{(-1)\times(-1)\times3}{3EI_\theta} + \frac{(1)\times(1)\times5}{3EI_\theta} = \frac{8}{3EI_\theta}$$

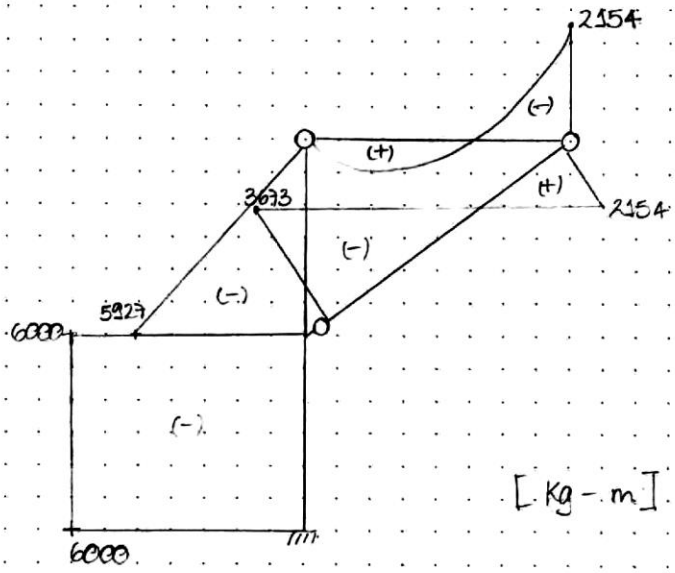
$$\begin{cases} 0 = 3400/EI_\theta + 8/EI_\theta \cdot X_1 - 5/6EI_\theta \cdot X_2 \quad (I) \\ 0 = 8000/EI_\theta - 5/6EI_\theta \cdot X_1 + 8/3EI_\theta \cdot X_2 \quad (II) \end{cases}$$

$X_1 = -2154$ Kg-m

$X_2 = -3673$ Kg-m

PREPARADOR: REDESCAR T. VIREL R.

DIAGRAMAS FINALES



[Kg - m]

PREPARADOR: RODRIGUEZ T. VIRGI 2