ESTEBAN TENREIRO & Profesionales asociados
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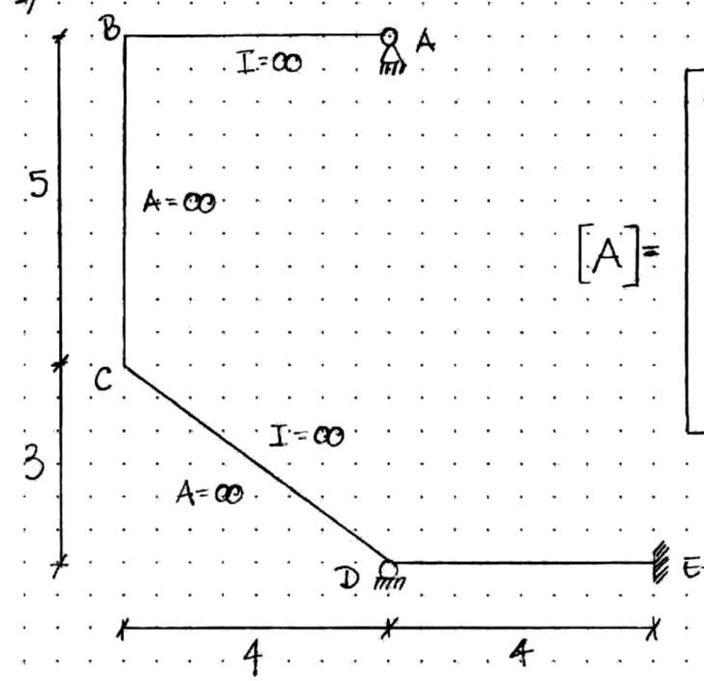
1)



$$[A]=\begin{bmatrix} -1 & 0 & 0 \\ 1/5 & -2/5 & -1/5 \\ 1/5 & -2/5 & -1/5 \\ 0 & -1/4 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$I.G. = I.G.^{3/R} - RESTRICCIONES \quad (I)$$

$$I.G.^{3/R} = \underset{(A)}{1} + \underset{(B)}{3} + \underset{(C)}{3} + \underset{(D)}{2} + \underset{(E)}{0} = 9$$

$$RESTRICCIONES = \underset{(A\text{-}B)}{2} + \underset{(B\text{-}C)}{1} + \underset{(C\text{-}D)}{3} + \underset{(D\text{-}E)}{0} = 6$$

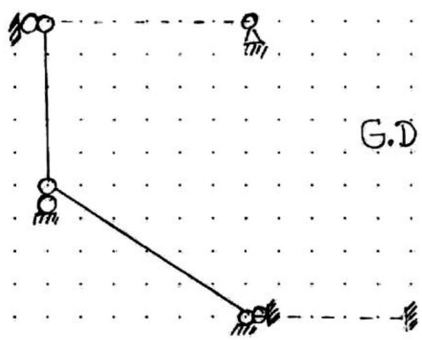
$$\boxed{I.G. = 9 - 6 = 3}$$

PREPARADOR: REDESCAR J. VIREL R.

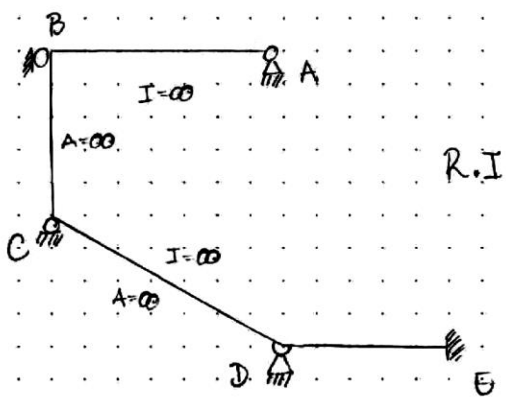
I.G. = G.D. + R.I. (II)

Para hallar los grados de desplazabilidad (G.D)

o IMAGEN CINEMÁTICA



G.D = 3



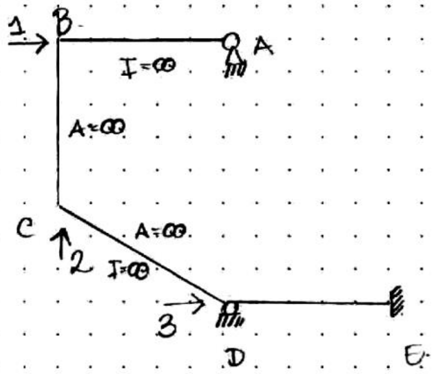
R.I. = 0

I.G. = 3 + 0 = 3

EN DEFINITIVA, TENEMOS 3 MODOS DE DESPLAZAMIENTO PLANTEADOS EN UN SISTEMA GLOBAL (Q-D); Y SUS RESPECTIVAS DEFORMACIONES DEFINIDAS EN EL SISTEMA LOCAL (q-d).

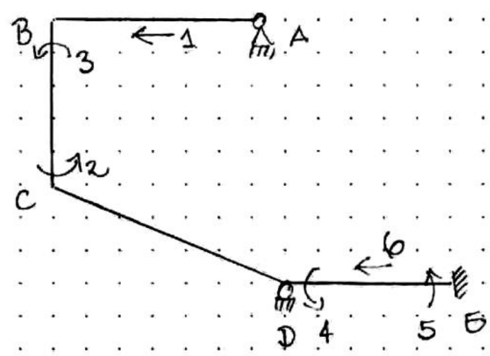
PREPARADOR: REDESBAR J. VIREL R.

∘ SISTEMA GLOBAL



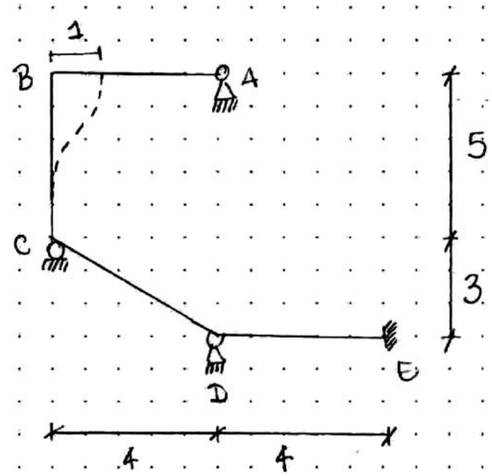
Q - D.

∘ SISTEMA LOCAL



q - d.

∘ Modo 1.



$\varphi_{BC} = -1/5$

$\theta_A = \theta_B = \theta_C = \theta_D = \theta_E = 0$

$\Delta L_{AB} = -1$

$\theta_{\overline{BC}} = \theta_B - \varphi_{BC}$

$\theta_{\overline{BC}} = 0 - (-1/5)$

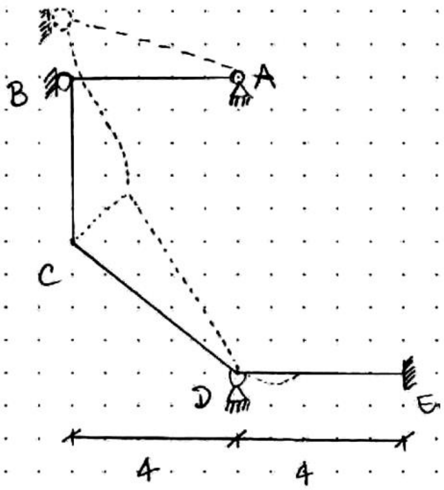
$\theta_{\overline{CB}} = \theta_C - \varphi_{CB}$

$\theta_{\overline{CB}} = 0 - (-1/5)$

$$d_{1i} \begin{cases} -1 \\ 1/5 \\ 1/5 \\ 0 \\ 0 \\ 0 \end{cases}$$

PREPARADOR: REDESCAR J. VIDEL Z.

∘ Modo 2



$\varphi_{AB} = -1/4$

$\delta_v^c = \varphi_{CD} \times dh_{pto}^{polo}$

$1 = \varphi_{CD} \times 4$

$\varphi_{CD} = -1/4$

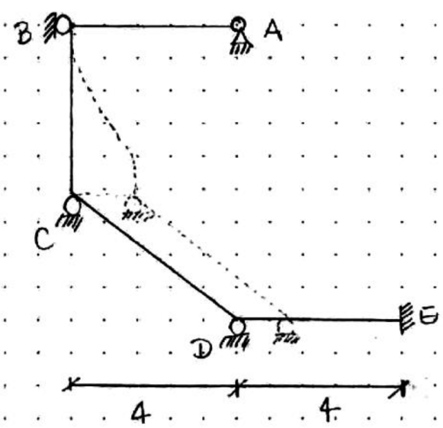
$\delta_h^c = 1/4 \times 3 = 3/4$

$\varphi_{BC} = \frac{\delta h^c}{dv_{pto}^{polo}} = \frac{3/4}{5} = \frac{3}{20}$

$$d2i \begin{cases} 0 \\ -1/4 - 3/20 \\ -1/4 - 3/20 \\ -1/4 \\ 0 \\ 0 \end{cases}$$

$\theta_A = \theta_B = -1/4$; $\theta_C = \theta_D = -1/4$

∘ Modo 3



$\varphi_{BC} = 1/5$

$\Delta L_{DE} = -1$

$\theta_{\bar{CB}} = \theta_C - \varphi_{CB}$

$\bar{\theta}_{CB} = 0 - (1/5)$

$\theta_{\bar{BC}} = \theta_B - \varphi_{BC}$

$\theta_{\bar{BC}} = 0 - (1/5)$

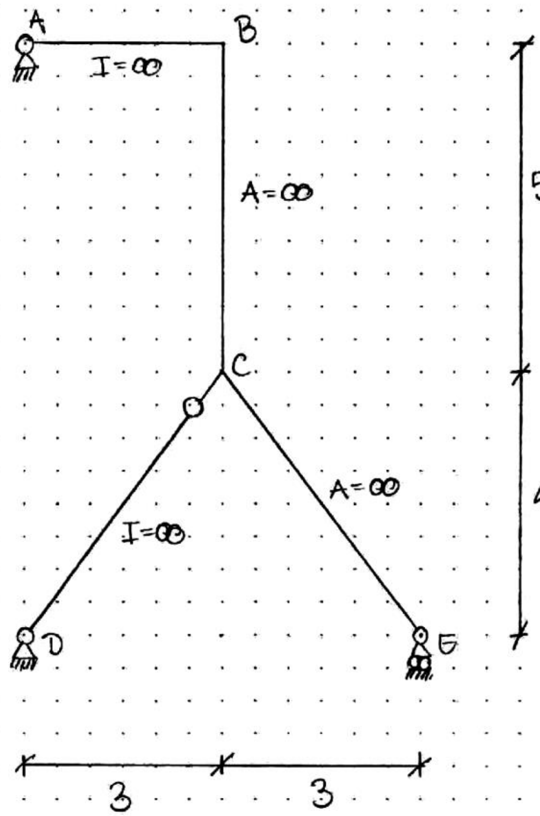
$$d3i \begin{cases} 0 \\ -1/5 \\ -1/5 \\ 0 \\ 0 \\ -1 \end{cases}$$

La barra CD se TRASLADA.

NOTA = Las desplazabilidades dadas a la estructura son unitarias.

PREPARADOR: REDEBCAR J. VIREL R.

2)



$$[A] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1/5 & 1/20 & -1/4 & 0 & 0 \\ 1/5 & -1/5 & 0 & 0 & 1 \\ 0 & 1/4 & -1/4 & 0 & 1 \\ 0 & 1/4 & -1/4 & 1 & 0 \\ 0 & 6/5 & -3/5 & 0 & 0 \end{bmatrix}$$

$I.G. = I.G^{S/R} - \text{RESTRICCIONES} \quad (I)$

$I.G.^{S/R} = \underset{(A)}{1} + \underset{(B)}{3} + \underset{(C)}{4} + \underset{(D)}{1} + \underset{(E)}{2} = 11$

$\text{RESTRICCIONES} = \underset{(A-B)}{2} + \underset{(B-C)}{1} + \underset{(C-E)}{1} + \underset{(C-D)}{2} = 6$

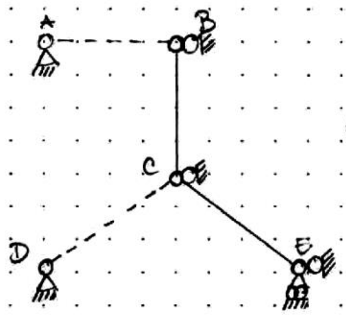
$$\boxed{I.G. = 11 - 6 = 5}$$

PREPARADOR: REDESCAR J. VIRLE R

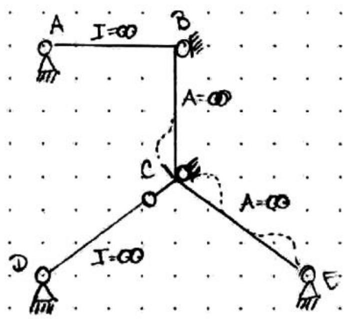
I.G = G.D. + R.I. (II)

Para hallar los grados de desplazabilidad:

° IMAGEN CINEMÁTICA



G.D = 3



R.I. = 2 (C↑ y E)

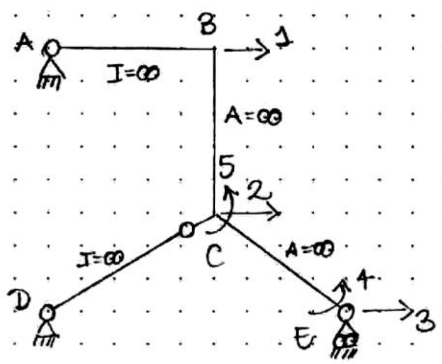
NOTA = C↑ indica que es la rotación de la parte superior de la rótula en C, puesto que la barra DC no tiene rotación independiente en el extremo en C. En consecuencia, en C hay dos posibles rotaciones, una del miembro CE y otra del miembro DC (esta última no es independiente).

$$I.G. = 3 + 2 = 5$$

TENEMOS 5 MODOS DE DESPLAZABILIDAD QUE DEFINIRÁN EL SISTEMA GLOBAL (Q-D) y SUS DEFORMACIONES RESPECTIVAS EN EL SISTEMA LOCAL (q-d).

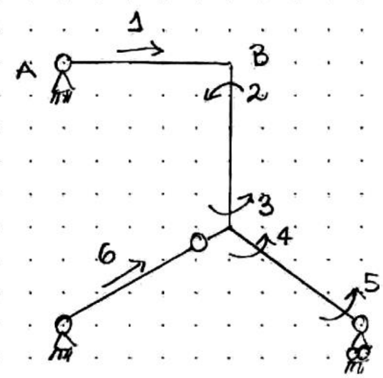
PREPARADOR: REDISEÑAR I VIRGI R

° SISTEMA GLOBAL



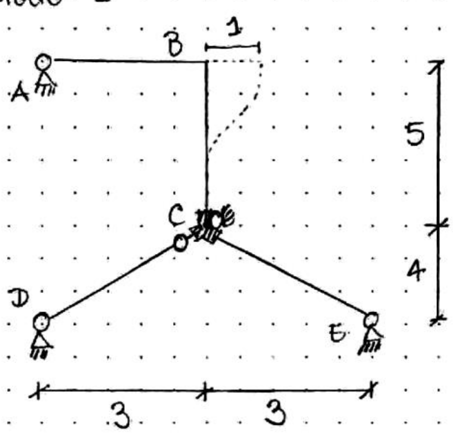
Q-D

° SISTEMA LOCAL



q-d

° Modo 1



$\Delta L_{AB} = 1$

$\psi_{BC} = -1/5$

$\bar{\theta}_{BC} = \theta_B - \psi_{BC}$

$\bar{\theta}_{BC} = 0 - (-1/5)$

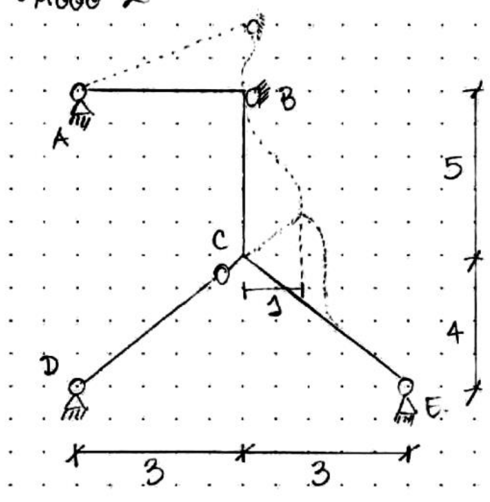
$\bar{\theta}_{CB} = \theta_C - \psi_{CB}$

$\bar{\theta}_{CB} = 0 - (-1/5)$

$$d_{1i} \begin{cases} 1 \\ 1/5 \\ 1/5 \\ 0 \\ 0 \\ 0 \end{cases}$$

Preparador: Rodriguez J Virel R

o Modo 2



$\varphi_{CE} = \dfrac{\delta^h_C}{dv^{PAO}_C} = \dfrac{1}{4}$

$\delta^v_C = \varphi_{CE} \times dh^{PAO}_C$

$\delta^v_C = \frac{1}{4} \times 3 = \frac{3}{4}$

$\delta^v_C = \delta^v_B$

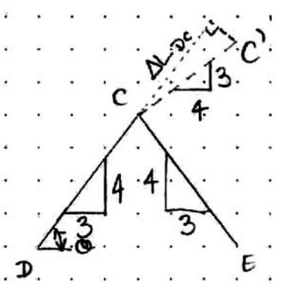
$\varphi_{AB} = \dfrac{\delta^v_B}{dh^{PAO}_B} = \dfrac{3/4}{3} = \frac{1}{4}$

$\varphi_{BC} = \dfrac{\delta^h_C}{dv^{PAO}_C} = \frac{1}{5}$

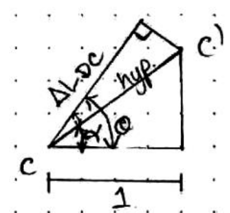
$$d_{2i} \begin{cases} 0 \\ \frac{1}{4} - \frac{1}{5} \\ 0 - \frac{1}{5} \\ 0 - (-\frac{1}{4}) \\ 0 - (-\frac{1}{4}) \\ \frac{6}{5} \end{cases}$$

$\theta_A = \theta_B = \varphi_{AB} = \frac{1}{4}$ — Por ser $I = \infty$

Para el calculo del ΔL_{DC}



En detalle



$\cos\theta = 3/5$

$\text{sen}\,\theta = 4/5$

$\cos\alpha = 4/5$

$\text{sen}\,\alpha = 3/5$

$\cos\alpha = 4/5 = 1/hyp \Rightarrow hyp = 5/4$

$\beta = \theta - \alpha$

$\cos\beta = \cos(\theta - \alpha)$

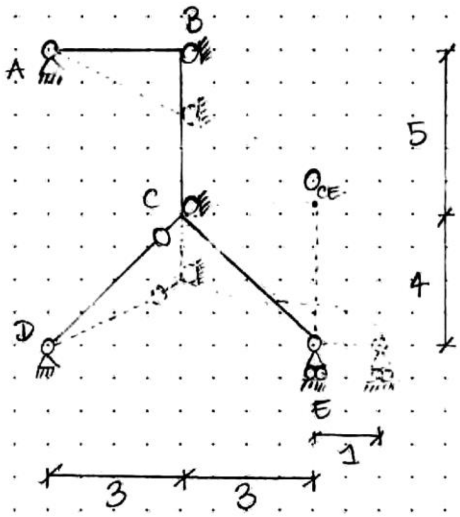
$\cos\beta = \cos\theta \times \cos\alpha + \text{sen}\,\alpha \times \text{sen}\,\theta$

$\cos\beta = \frac{3}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{4}{5} = \frac{24}{25} = \frac{\Delta L_{DC}}{hyp}$

$\Delta L_{DC} = \frac{24}{25} \times \frac{5}{4} = \frac{6}{5}$

Preparador: Redescar J. Virel R.

- Modo 3.



$$\varphi_{CE} = \frac{\delta^h_E}{d^{polo}_{vE}} = \frac{1}{4}$$

$$\delta^v_C = \varphi_{CE} \times d^{polo}_{hC}$$

$$\delta^v_C = \frac{1}{4} \times 3 = \frac{3}{4}$$

$$\delta^v_C = \delta^v_B$$

$$\varphi_{AB} = \frac{\delta^v_B}{d^{polo}_{hB}} = \frac{3/4}{3} = -\frac{1}{4}$$

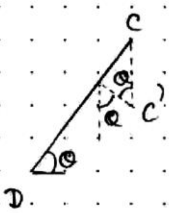
$$\underbrace{\theta_A = \theta_B = -\frac{1}{4} = \varphi_{AB}}$$

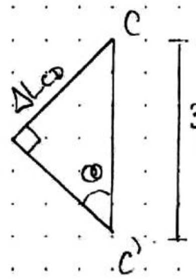
Por ser miembro de $I = \infty$.

$$d_{3i} \begin{cases} 0 \\ -1/4 \\ 0 \\ 0-(1/4) \\ 0-(1/4) \\ -3/5 \end{cases}$$

- Para estimar el ΔL_{CD}.

En detalle



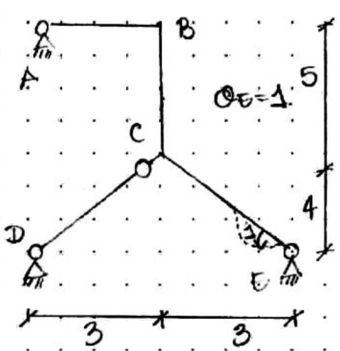


$$\text{sen}\,\theta = 4/5$$

$$\text{sen}\,\theta = 4/5 = \frac{\Delta L_{CD}}{3/4}$$

$$\Delta L_{CD} = \frac{4}{5} \times \frac{3}{4} = -3/5$$

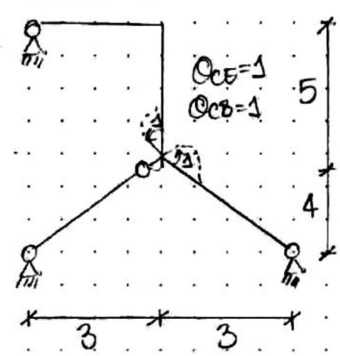
- Modo 4.



$$\theta_E = 1$$

$$d_{4i} \begin{cases} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{cases}$$

- Modo 5.



$$\theta_{CE} = 1$$
$$\theta_{CB} = 1$$

$$d_{5i} \begin{cases} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{cases}$$

PREPARADOR: REDIBCAR. J. VIREL R.

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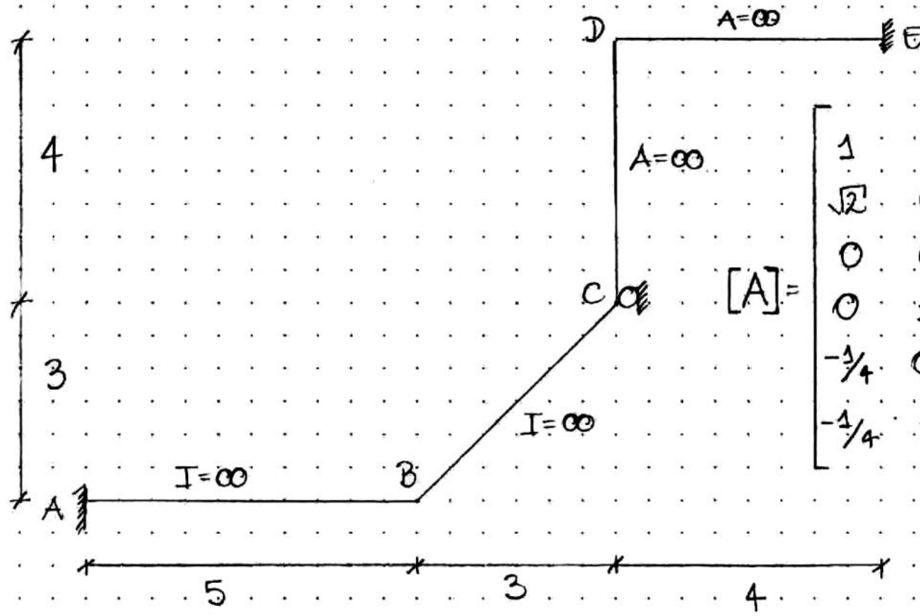
3)



$$[A]=\begin{bmatrix} 1 & 0 \\ \sqrt{2} & 0 \\ 0 & 0 \\ 0 & 1 \\ -1/4 & 0 \\ -1/4 & 1 \end{bmatrix}$$

$I.G. = I.G^{S/R} - \text{RESTRICCIONES} \quad (I)$

$I.G^{S/R} = \underset{(B)}{3} + \underset{(C)}{2} + \underset{(D)}{3} = 8$

$\text{RESTRICCIONES} = \underset{(A-B)}{2} + \underset{(B-C)}{2} + \underset{(C-D)}{1} + \underset{(D-E)}{1} = 6$

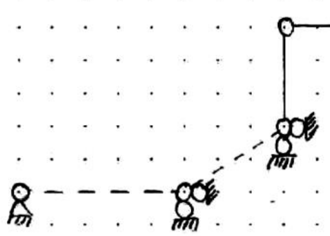
$\boxed{I.G. = 8 - 6 = 2}$

PREPARADOR: REDEBSAR J. VIREL R

I.G. = G.D. + R.I. (II)

Para hallar los Grados de Desplazabilidad (G.D.).

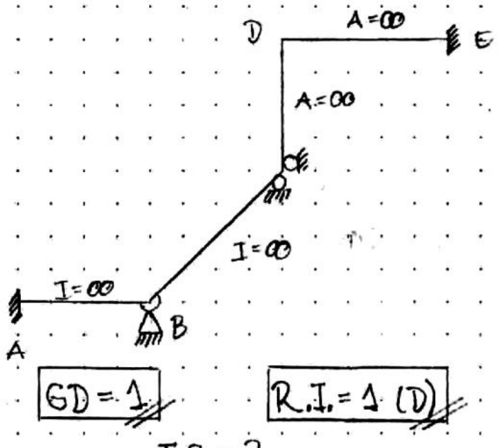
o IMAGEN CINEMÁTICA.



G.D. = 3

Los grados de desplazabilidad estimados superan el número de indeterminaciones geométricas halladas en (I), en consecuencia, es probable que parte de las desplazabilidades estimadas no existan debido a la presencia de hipótesis restrictivas de deformaciones (A=∞ y/ó I=∞).

o Revisando la estructura restringiendo las posibles desplazabilidades estimadas en (II) se tiene que:



Se identifica que existe dependencia entre el desplazamiento vertical de "B" y de "C", en vista que se debe mantener el ángulo existente entre las barras AB y BC, puesto que ambas poseen I=∞ y su rotación será la misma, como si se tratase de una única barra.

Además, la condición de empotramiento en "A" obliga a que el desplazamiento horizontal y vertical de "B" sean dependientes entre sí para cumplir con la condición de I=∞.

En conclusión, los grados de desplazabilidad disminuyen de 3 a 1.

PREPARADOR: REDESCAR J. VIRLA R.

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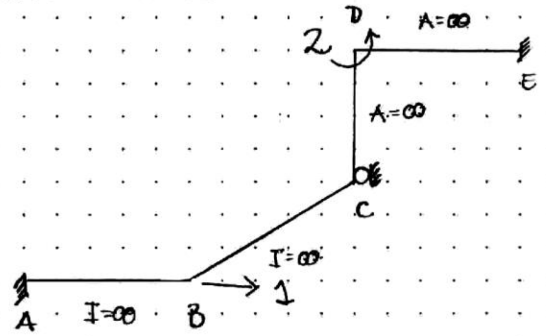
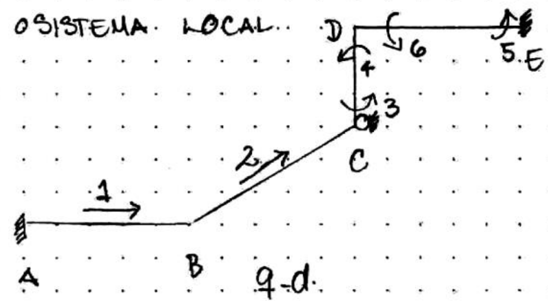
o SISTEMA GLOBAL



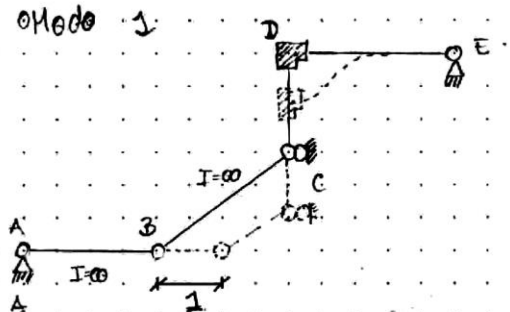
Q - D

o SISTEMA LOCAL



q - d

o Modo 1

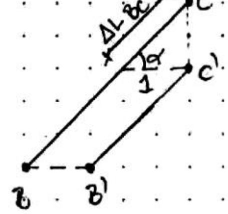


$\Delta L_{AB} = 1$

$\delta_D^v = \delta_C^v = 1$

$\psi_{DE} = \frac{1}{4}$

$$d_{1i} \begin{cases} 1 \\ \sqrt{2} \\ 0 \\ 0 \\ -1/4 \\ -1/4 \end{cases}$$



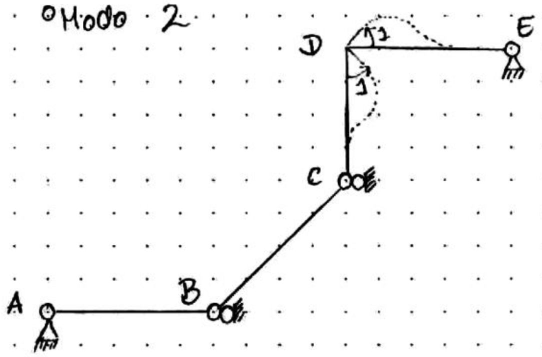
$\cos\alpha = \frac{\sqrt{2}}{2}$ $(\alpha = 45°)$

$\cos\alpha = \frac{\sqrt{2}}{2} = \frac{1}{\Delta L_{BC}} \Rightarrow \Delta L_{BC} = \sqrt{2}$

NOTA = Las desplazabilidades asociadas a cada modo son unitarias

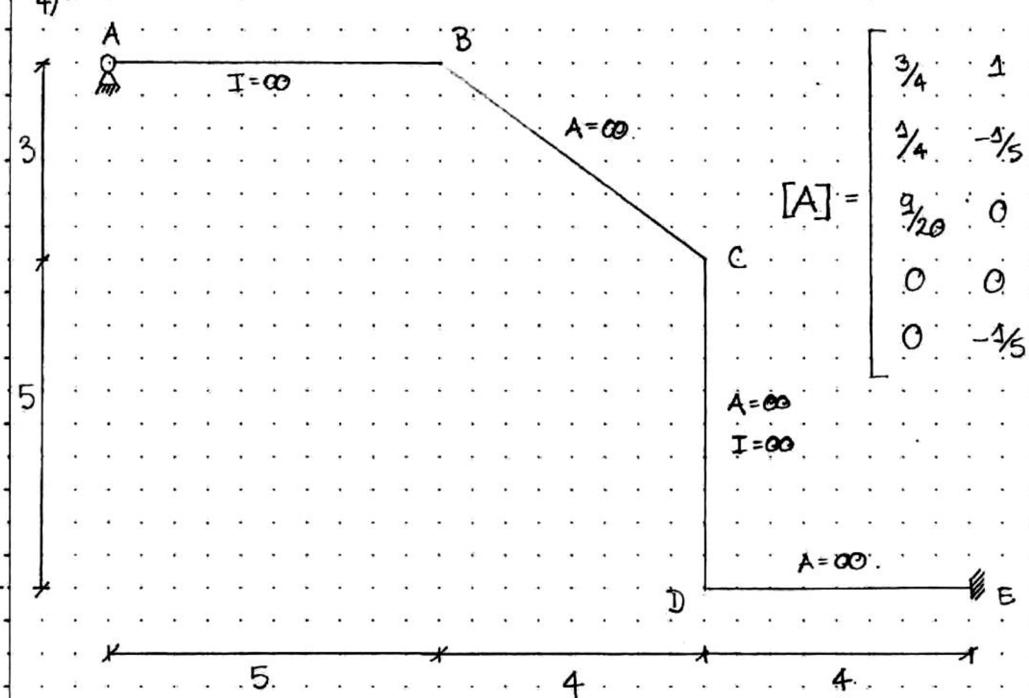
PREPARADOR: REDESCAR J. VIREL R.

° Modo 2



$$d_{i2}\begin{Bmatrix}0\\0\\0\\1\\0\\1\end{Bmatrix}$$

4)



$$[A] = \begin{bmatrix} 3/4 & 1 & -3/4 \\ 1/4 & -1/5 & -1/4 \\ 9/20 & 0 & -1/4 \\ 0 & 0 & 1/4 \\ 0 & -1/5 & 1/4 \end{bmatrix}$$

$I.G. = I.G^{S/R} - \text{RESTRICCIONES} \quad (I)$

$I.G.^{S/R} = \underset{(A)}{1} + \underset{(B)}{3} + \underset{(C)}{3} + \underset{(D)}{3} = 10$

$\text{RESTRICCIONES} = \underset{(A\text{-}B)}{2} + \underset{(B\text{-}C)}{1} + \underset{(C\text{-}D)}{3} + \underset{(D\text{-}E)}{1} = 7$

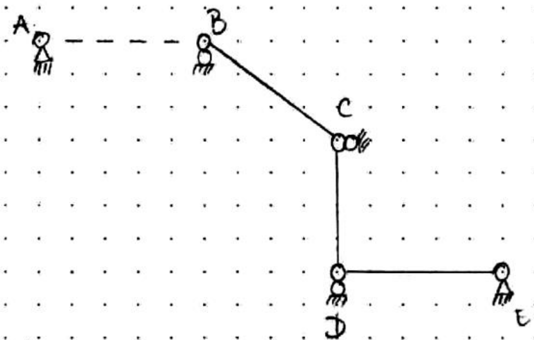
$\boxed{I.G. = 10 - 7 = 3}$

PREPARADOR: REDEBEAR J. VIREL R.

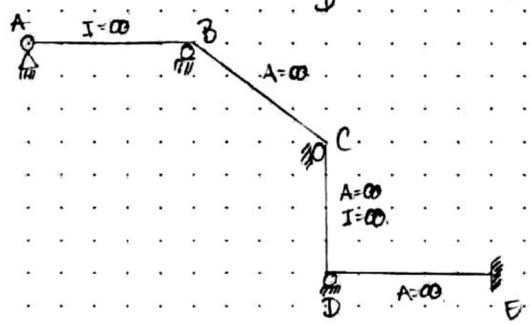
$$I.G. = G.D. + R.I. \quad (II)$$

Para hallar los Grados de Desplazabilidad (G.D.)

∘ IMAGEN CINEMÁTICA



G.D. = 3

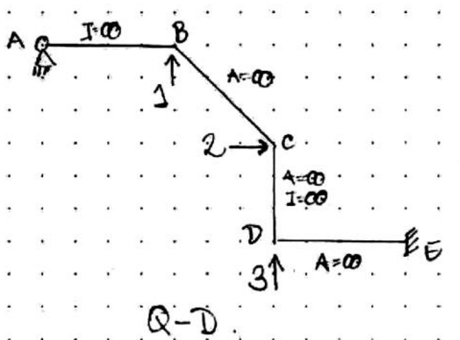


R.I. = 0

$$\boxed{I.G. = 3 + 0 = 3}$$

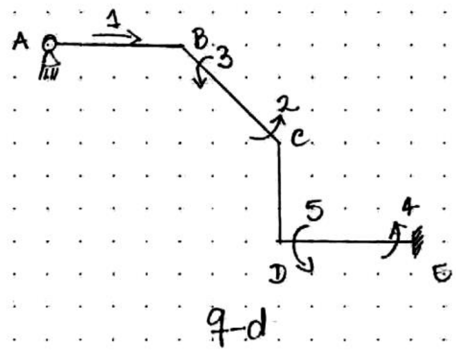
En conclusión, se tiene una estructura con tres modos de desplazamiento capaces de generar deformaciones sobre el sistema.

∘ SISTEMA GLOBAL



Q-D

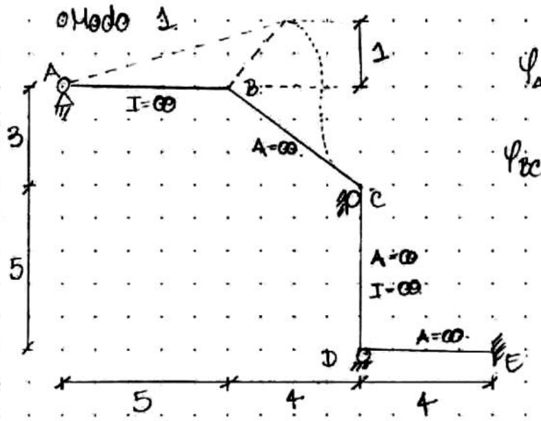
∘ SISTEMA LOCAL



q-d

PREPARADOR: RENZO J. VIREL R.

∘ Modo 1.



$\varphi_{AB} = 1/5$

$\varphi_{BC} = \dfrac{\delta_B^v}{dh_{B-POLO}} = -1/4$

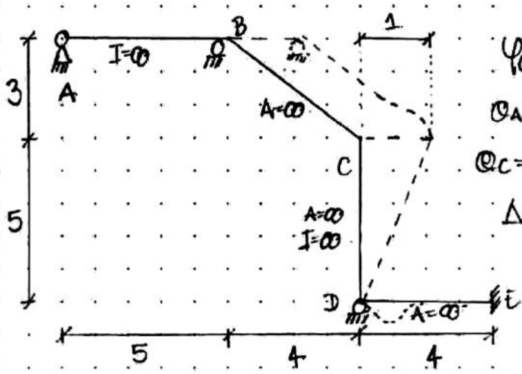
$\Delta L^{AB} = \delta_B^h = \varphi_{BC} \times dv_{B-POLO}$

$\Delta L^{AB} = 1/4 \times 3 = 3/4$

$d1i \begin{cases} 3/4 \\ 0-(-1/4) \\ 1/5-(-1/4) \\ 0 \\ 0 \end{cases}$

$\Theta_A = \Theta_B = \varphi_{AB} = 1/5 \quad \Theta_C = \Theta_D = \Theta_E = 0$

∘ Modo 2.



$\varphi_{CD} = -1/5$

$\Theta_A = \Theta_B = \Theta_E = 0$

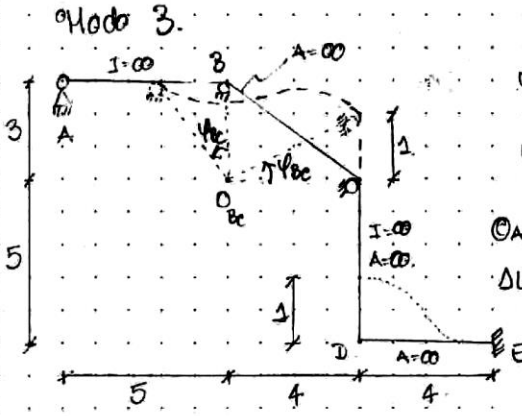
$\Theta_C = \Theta_D = -1/5$

$\Delta L^{AB} = 1$

$d2i \begin{cases} 1 \\ -1/5-(0) \\ 0 \\ 0 \\ -1/5-(0) \end{cases}$

La barra BC se traslada.

∘ Modo 3.



$\varphi_{DE} = -1/4$

$\varphi_{BC} = 1/4$

$\Theta_A = \Theta_B = \Theta_C = \Theta_D = \Theta_E = 0$

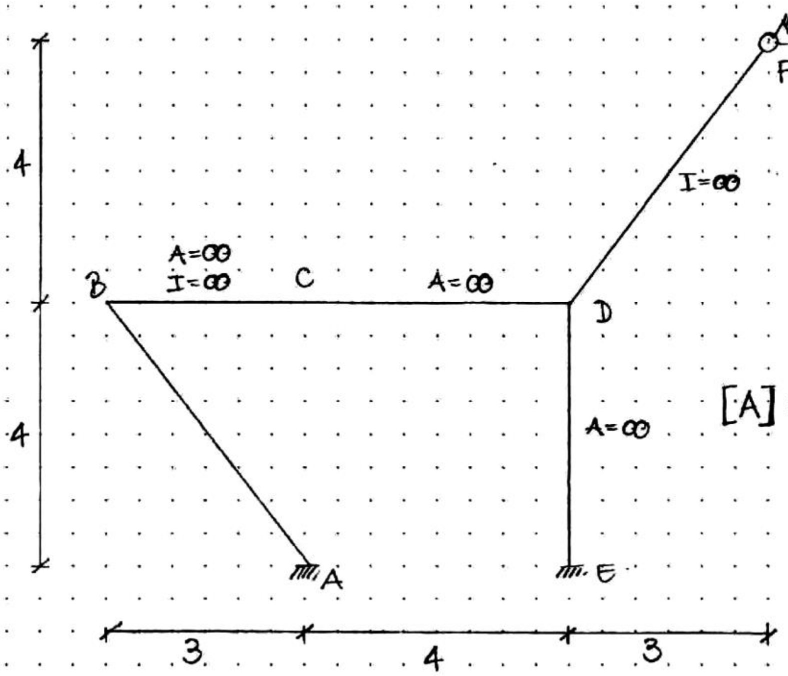
$\Delta L_{AB} = \delta_B^h = \varphi_{BC} \times dv_{B-POLO}$

$\Delta L_{AB} = -1/4 \times 3 = -3/4$

$d3i \begin{cases} -3/4 \\ 0-1/4 \\ 0-1/4 \\ 0-(-1/4) \\ 0-(-1/4) \end{cases}$

PREPARADOR: RIDELCAR J. VIREL R.

5)



$$[A] = \begin{bmatrix} 3/25 & 0 & 4/25 \\ 3/25 & 1/3 & 4/25 \\ 4/5 & 0 & -3/5 \\ 0 & 1/4 & 4/25 \\ -1/3 & 7/12 & 0 \\ 0 & 0 & 1/4 \\ 0 & 0 & 41/100 \\ 0 & 0 & -3/5 \end{bmatrix}$$

$$I.G. = I.G^{S/R} - \text{RESTRICCIONES} \quad (I)$$

$$I.G^{S/R} = \underset{(B)}{3} + \underset{(C)}{3} + \underset{(D)}{3} + \underset{(F)}{1} = 10$$

$$\text{RESTRICCIONES} = \underset{(B-C)}{3} + \underset{(C-D)}{1} + \underset{(D-E)}{1} + \underset{(D-F)}{2} = 7$$

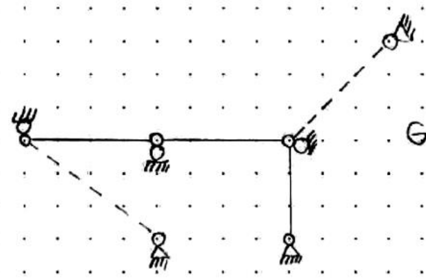
$$\boxed{I.G. = 10 - 7 = 3}$$

PREPARADOR: REDEBCAZ J. VIREL R

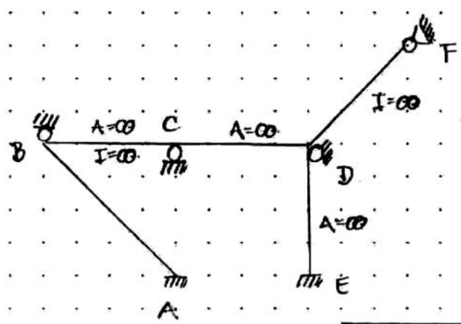
I.G. = G.D. + R.I. (II)

Para hallar los Grados de Desplazabilidad.

- IMAGEN CINEMÁTICA



G.D. = 3



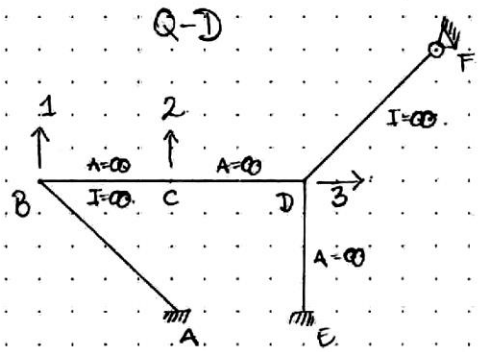
R.I. = 0

I.G. = 3 + 0 = 3

La Estructura posee tres modos de desplazamiento, en consecuencia se define el sistema global y local.

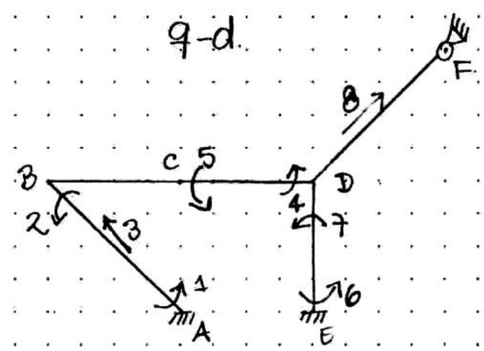
- SISTEMA GLOBAL

Q–D



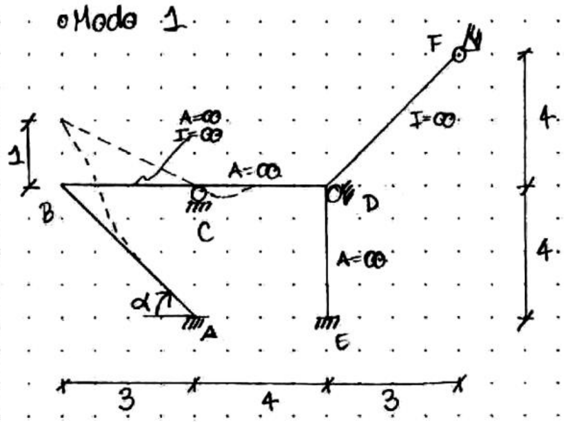
- SISTEMA LOCAL

q–d



PREPARADOR: REDIBEAR J. VIRA R.

• Modo 1

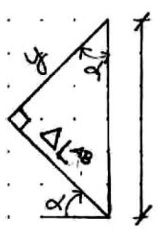


$\psi_{BC} = -1/3$

$\theta_A = \theta_D = \theta_E = \theta_F = 0$

$\theta_B = \theta_C = -1/3$

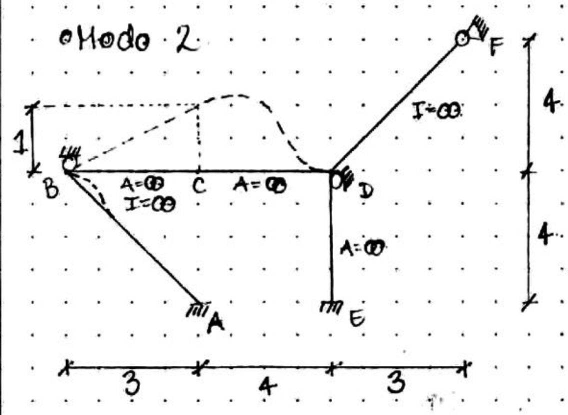
$$d_{1i} \begin{cases} 0-(-3/25) \\ 0-(-3/25) \\ 4/5 \\ 0 \\ -1/3-(0) \\ 0 \\ 0 \\ 0 \end{cases}$$



$\operatorname{sen}\alpha = \frac{4}{5} = \frac{\Delta L^{AB}}{1} \Rightarrow \Delta L^{AB} = 4/5$

$\cos\alpha = \frac{3}{5} = \frac{y}{1} \Rightarrow y = 3/5 \Rightarrow \psi_{AB} = y/5 = \frac{3/5}{5} = -3/25$

• Modo 2



$\psi_{BC} = 1/3$

$\psi_{CD} = -1/4$

$\theta_B = \theta_C = 1/3$

$\theta_A = \theta_D = \theta_E = \theta_F = 0$

$$d_{2i} \begin{cases} 0 \\ 1/3-(0) \\ 0 \\ 0-(-1/4) \\ 1/3-(-1/4) \\ 0 \\ 0 \\ 0 \end{cases}$$

NOTA = cada desplazabilidad asociada a cada modo es unitaria

PREPARADOR: REDESBAR J. VIREL R.

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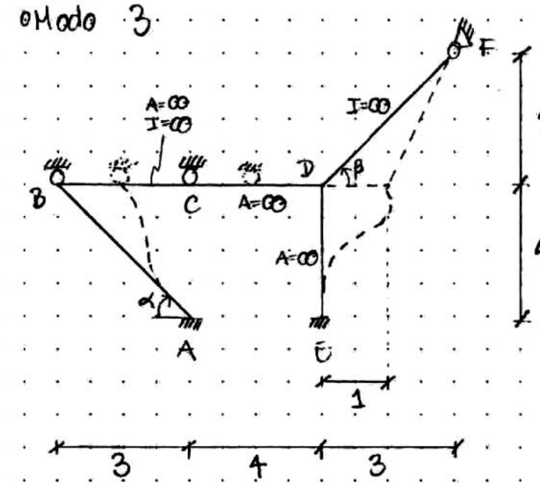
o Modo 3.



$\theta_B = \theta_C = 0$

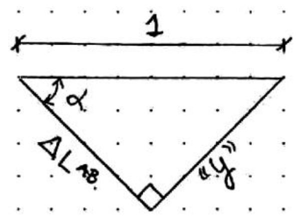
$\theta_D = \theta_F = 4/25$

$\theta_A = \theta_E = 0$

$\varphi_{DE} = -1/4$

$$d3i \left\{ \begin{array}{l} 0-(-4/25) \\ 0-(-4/25) \\ -3/5 \\ 4/25-(0) \\ 0 \\ 0-(-1/4) \\ 4/25-(-1/4) \\ -3/5 \end{array} \right.$$

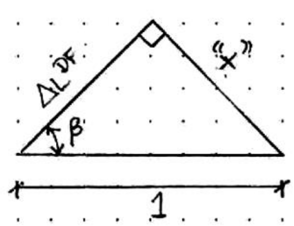
Para hallar el ΔL^{AB}



$\cos\alpha = 3/5 = -\Delta L_{AB} \Rightarrow \Delta L_{AB} = -3/5$

$\operatorname{sen}\alpha = 4/5 = y \Rightarrow \varphi_{AB} = y/5 = -4/25$

Para hallar el ΔL^{DF}

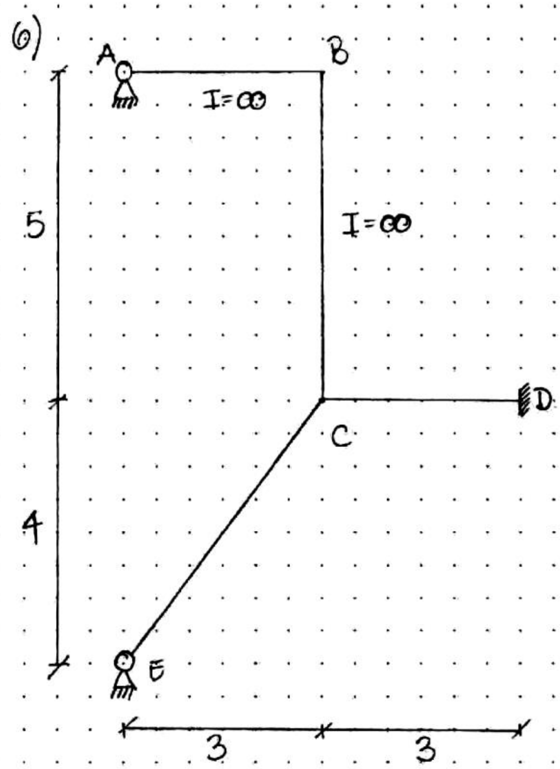


$\cos\beta = 3/5 = -\Delta L_{DF} \Rightarrow \Delta L_{DF} = -3/5$

$\operatorname{sen}\beta = 4/5 = x \Rightarrow \varphi_{DF} = \frac{x}{5} = 4/25$

PREPARADOR: RIDDEBCAR J. VIREL R.

6)



$$[A] = \begin{bmatrix} -5/3 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 1/3 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/3 & -3/25 & 4/25 & 0 \\ 0 & -3/25 & 4/25 & 1 \\ 0 & -4/5 & 3/5 & 0 \end{bmatrix}$$

$$I.G. = I.G.^{S/R} - \text{RESTRICCIONES} \quad (I)$$

$$I.G^{S/R} = \underset{(A)}{1} + \underset{(B)}{3} + \underset{(C)}{3} + \underset{(E)}{1} = 8$$

$$\text{RESTRICCIONES} = \underset{(A\text{-}B)}{2} + \underset{(B\text{-}C)}{2} = 4$$

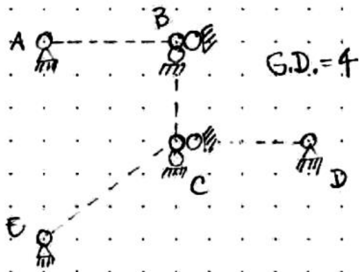
$$\boxed{I.G. = 8 - 4 = 4}$$

PREPARADOR: REDESCAR J. VIRCL R.

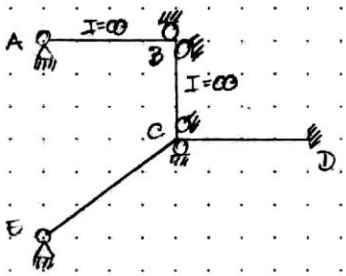
I.G. = G.D. + R.I. (II)

Para hallar los Grados de Desplazabilidad

○ IMAGEN CINEMÁTICA



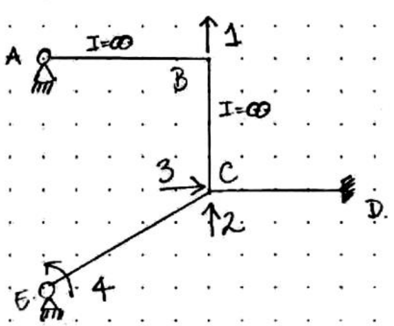
Por inspección, se identifica al menos una rotación independiente en E, en consecuencia, el número de I.G. supera a las halladas en (I), por lo que la estructura será revisada a continuación:



Para mantener la condición de I=∞ en las barras AB y BC, deberá existir dependencia entre el desplazamiento horizontal y vertical en B, puesto que ambas barras rotarán lo mismo. Los G.D. se reducen entonces de 4 a 3.

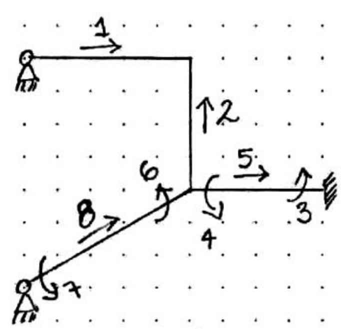
Definiendo los sistemas global y local:

○ SISTEMA GLOBAL



Q - D

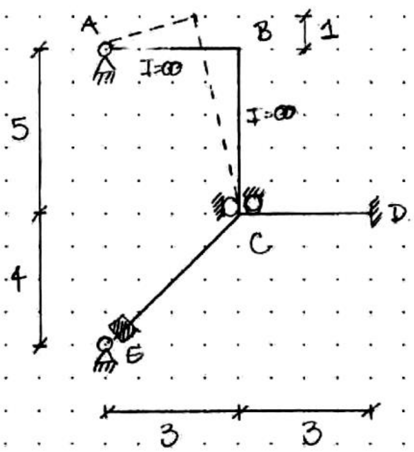
○ SISTEMA LOCAL



q - d

PREPARADOR: REVERCAR J. VIRAL R.

• Modo 1.



$\varphi_{AB} = \varphi_{BC} = 1/3$

$\theta_A = \theta_B = \theta_C = 1/3$

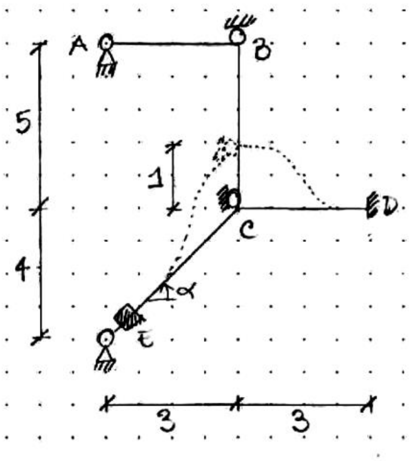
$\theta_D = \theta_E = 0$

$\delta^h_B = \Delta L^{AB} = \varphi_{BC} \times d_{V_{B\text{-}C}} = 1/3 \times 5$

$\Delta L^{AB} = -5/3$

$$d_{1i} \begin{cases} -5/3 \\ 1 \\ 1/3 \\ 0 \\ 0 \\ 1/3 \\ 0 \\ 0 \end{cases}$$

• Modo 2.



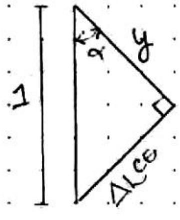
$\varphi_{CD} = -1/3$

$\theta_i = 0$

$\Delta L_{BC} = -1$

$$d_{2i} \begin{cases} 0 \\ -1 \\ 0 \\ 0-(-1/3) \\ 0 \\ 0-3/25 \\ 0-3/25 \\ -4/5 \end{cases}$$

Para hallar el ΔL_{CE}

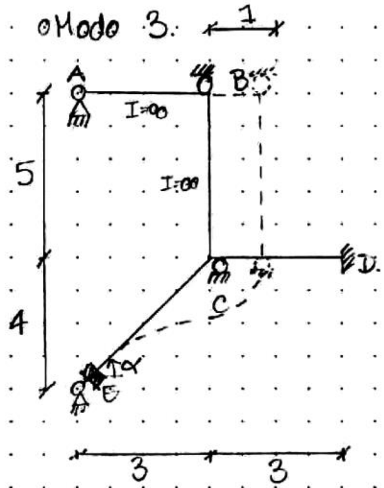


$\text{sen}\,\alpha = 4/5 = |\Delta L_{CE}| \Rightarrow \Delta L_{CE} = -4/5$

$\cos\alpha = 3/5 = y \Rightarrow \varphi_{CE} = y/5 = 3/25$

PREPARADOR: FEDERICAR J. VIRLA R.

○ Modo 3.

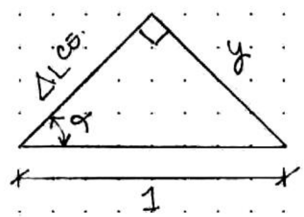


$\Delta L_{AB} = 1$

$\theta i = 0$

$$d_{3i} \begin{cases} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0-(-4/25) \\ 0-(-4/25) \\ 3/5 \end{cases}$$

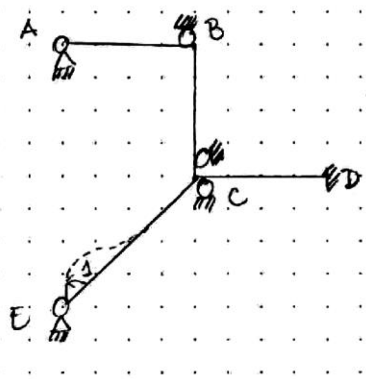
Para hallar el ΔL^{CE}



$\cos\alpha = 3/5 = \Delta L^{CE}$

$\operatorname{sen}\alpha = 4/5 = y \Rightarrow \psi_{CE} = -y/5 = -4/25$

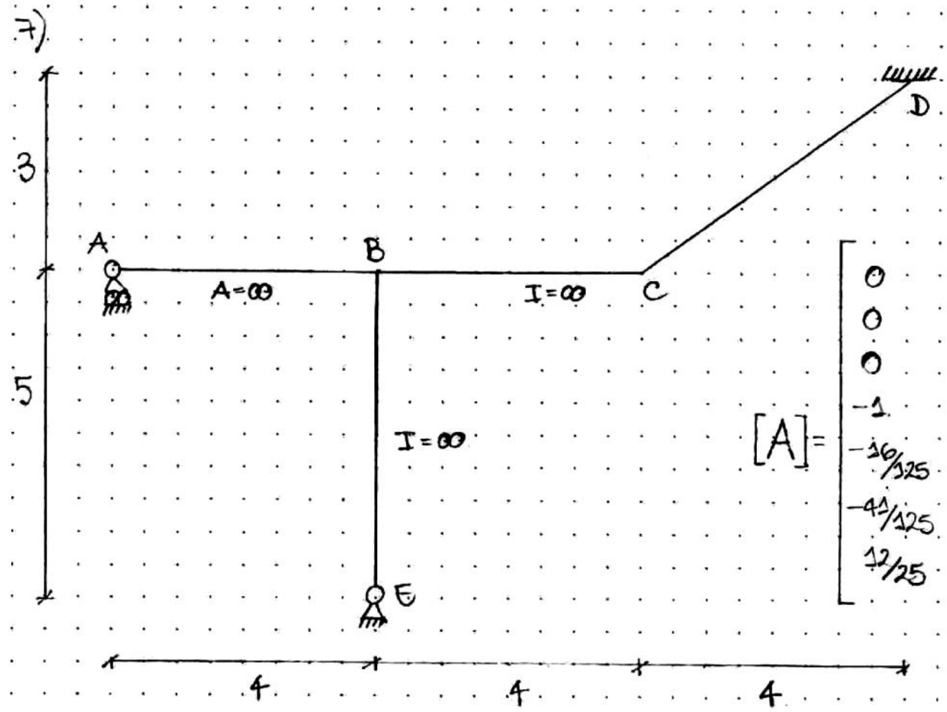
○ Modo 4



$$d_{4i} \begin{cases} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{cases}$$

PREPARADOR: REDEZCAR J. VIREL R.

7).



$$[A]=\begin{bmatrix} 0 & -3/4 & 0 & 0 \\ 0 & -1/4 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -16/125 & 4/25 & -3/25 & 0 \\ -41/125 & 4/25 & -3/25 & 0 \\ 12/25 & -3/5 & -4/5 & 0 \end{bmatrix}$$

$$I.G. = I.G^{SR} - \text{RESTRICCIONES} \quad (I)$$

$$I.G^{SR} = \underset{(A)}{2} + \underset{(B)}{3} + \underset{(C)}{3} + \underset{(E)}{1} = 9$$

$$\text{RESTRICCIONES} = \underset{(A-B)}{1} + \underset{(B-C)}{2} + \underset{(B-E)}{2} = 5$$

$$\boxed{I.G. = 9 - 5 = 4}$$

PREPARADOR: REDESCAR J. VIREL R.

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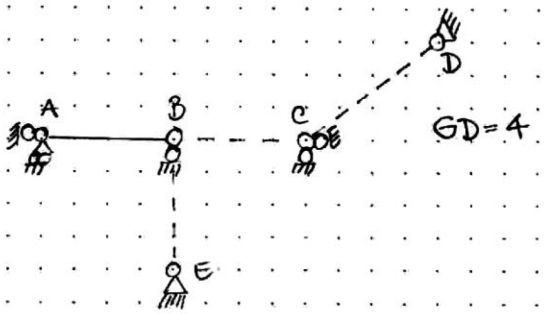
I.G. = G.D. + R.I. (II)

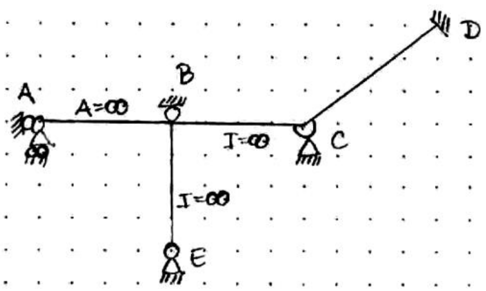
Para hallar los Grados de Desplazabilidad:

∘ IMAGEN CINEMÁTICA.



Se ha identificado al menos una rotación independiente en A (por inspección), por lo que al sumar los G.D, el número de I.G. superará las halladas en (I) y ambas ecuaciones no coincidirán. La estructura debe ser revisada.

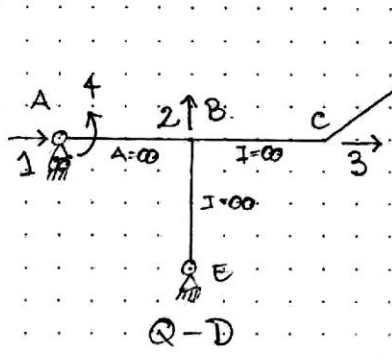
Revisando la estructura:



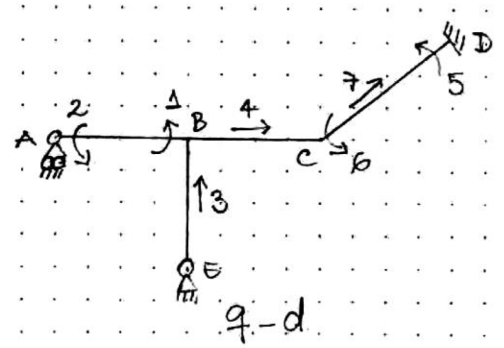
Para mantener el ángulo existente entre las barras BE y BC (90°), existirá dependencia entre el desplazamiento vertical de B y de C, por lo que los grados de desplazabilidad se reducen de 4 a 3.

Definiendo los sistemas global y local:

∘ SISTEMA GLOBAL

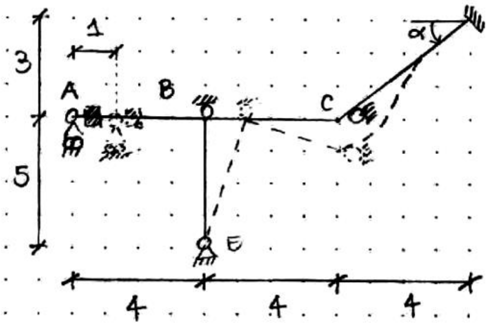


∘ SISTEMA LOCAL



PREPARADOR: REDESCAR J. VIREL R.

∘ Modo I



$\varphi_{BE} = \frac{1}{5}$

$\Delta L_{BC} = -1$

$\Delta L_{BE} = 0$

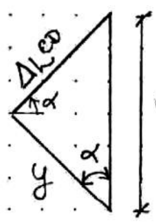
$\theta_E = \theta_B = \theta_C = -\frac{1}{5}$

$\theta_A = \theta_D = 0$

$$d_{1i} \begin{cases} 0 \\ 0 \\ 0 \\ -1 \\ 0 - \left(\frac{16}{125}\right) \\ -\frac{1}{5} - \left(\frac{16}{125}\right) \\ \frac{12}{25} \end{cases}$$

Para hallar el ΔL_{CD}.

$\delta_C^V = \varphi_{BE} \times dh_{C\text{-}polo} \quad \frac{1}{5} \times 4 = \frac{4}{5}$



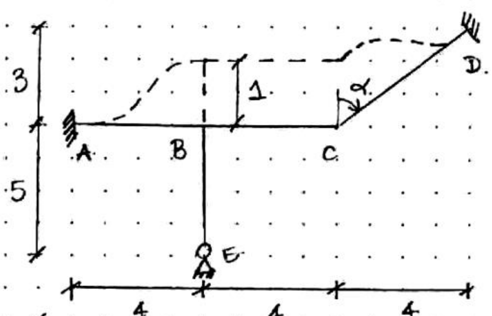
$\delta_C^V = \frac{4}{5}$

$\operatorname{sen}\alpha = \frac{3}{5} = \frac{\Delta L^{CD}}{\frac{4}{5}} \Rightarrow \Delta L^{CD} = \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$

$\cos\alpha = \frac{4}{5} = \frac{y}{\frac{4}{5}} \Rightarrow y = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$

$\varphi_{CD} = \frac{y}{5} = \frac{\frac{16}{25}}{5} = \frac{16}{125}$

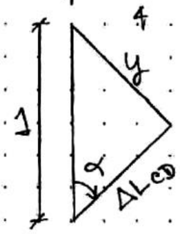
∘ Modo II



$\varphi_{AB} = +\frac{1}{4}$

$\theta_i = 0$

$$d_{2i} \begin{cases} 0 - \left(\frac{1}{4}\right) \\ 0 - \left(\frac{1}{4}\right) \\ 1 \\ 0 \\ 0 - \left(-\frac{4}{25}\right) \\ 0 - \left(-\frac{4}{25}\right) \\ -\frac{3}{5} \end{cases}$$

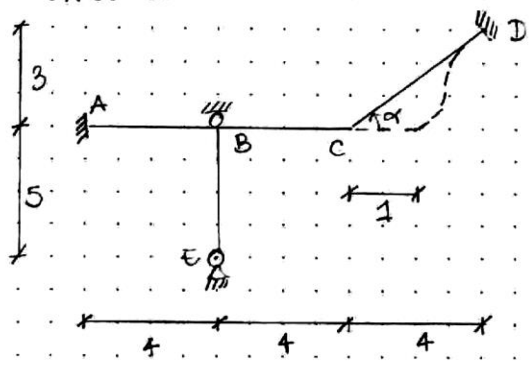


$\cos\alpha = \frac{3}{5} = |\Delta L^{CD}| \Rightarrow \Delta L_{CD} = -\frac{3}{5}$

$\operatorname{sen}\alpha = \frac{4}{5} = y \Rightarrow \varphi_{CD} = \frac{y}{5} = -\frac{4}{25}$

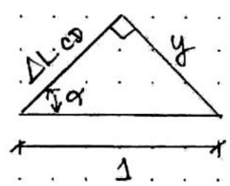
Preparador: Redesbar J. Viree 2.

• Modo III



$$d3i \begin{cases} 0 \\ 0 \\ 0 \\ 1 \\ 0 \; -3/25 \\ 0 \; -3/25 \\ -4/5 \end{cases}$$

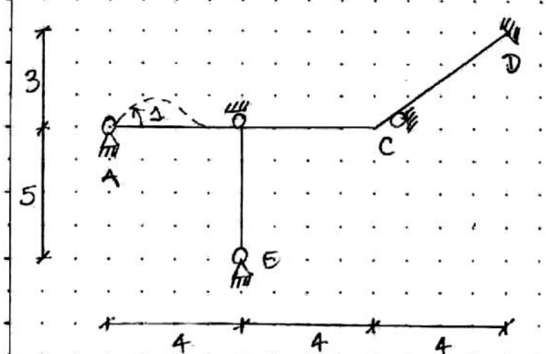
Para hallar el ΔL^{CD}



$\cos\alpha = 4/5 = |\Delta L_{CD}| \Rightarrow \Delta L_{CD} = -4/5$

$\text{sen}\,\alpha = 3/5 = y \Rightarrow \varphi_{CD} = \frac{3/5}{5} = 3/25$

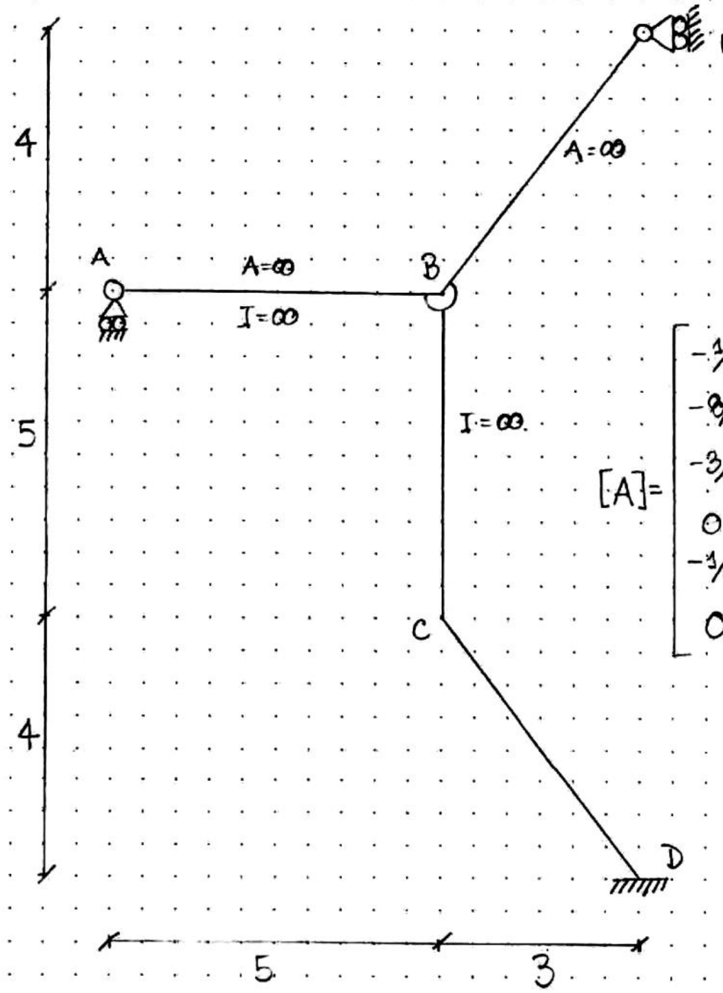
• Modo IV



$$d4i \begin{cases} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{cases}$$

PREPARADOR: REDESCAR T. VREL R.

8)



$$[A]=\begin{bmatrix} -\frac{1}{4} & 0 & 0 & 0 & 1 \\ -\frac{3}{20} & -\frac{1}{5} & 0 & 0 & 0 \\ -\frac{3}{4} & -1 & 0 & 1 & 0 \\ 0 & 0 & \frac{4}{25} & -\frac{3}{25} & 0 \\ -\frac{1}{5} & 0 & \frac{3}{25} & -\frac{3}{25} & 0 \\ 0 & 0 & -\frac{3}{5} & -\frac{4}{5} & 0 \end{bmatrix}$$

$$I.G. = I.G^{S/R} - \text{RESTRICCIONES (I)}$$

$$I.G.^{S/R} = \underset{(A)}{2} + \underset{(B)}{4} + \underset{(C)}{3} + \underset{(E)}{2} = 11$$

$$\text{RESTRICCIONES} = \underset{(A-B)}{3} + \underset{(B-E)}{1} + \underset{(B-C)}{2} = 6$$

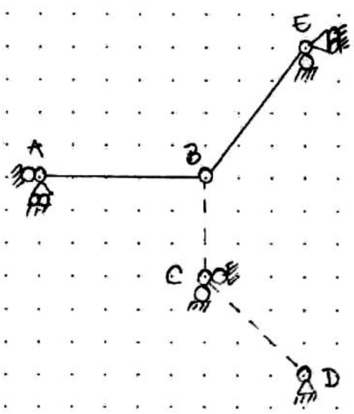
$$\boxed{I.G. = 11 - 6 = 5}$$

PREPARADOR: REDESCAR I. VIREL R.

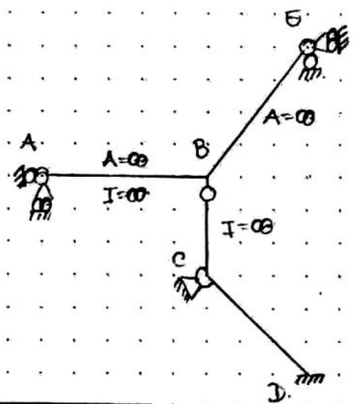
$I.G. = G.D. + R.I.$ (II)

Para hallar los Grados de Desplazabilidad:

∘ IMAGEN CINEMÁTICA.



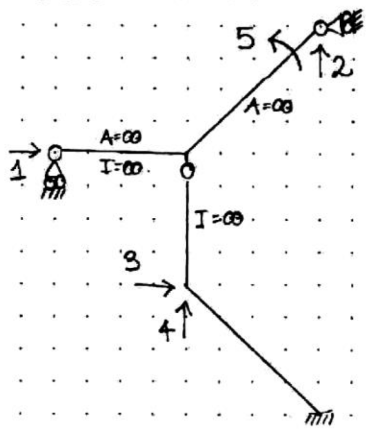
GD = 4



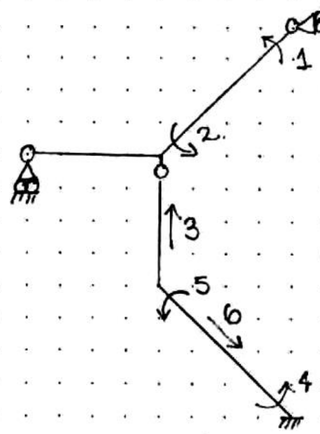
R.I. = 1 (E)

$I.G. = 4 + 1 = 5$

∘ SISTEMA GLOBAL



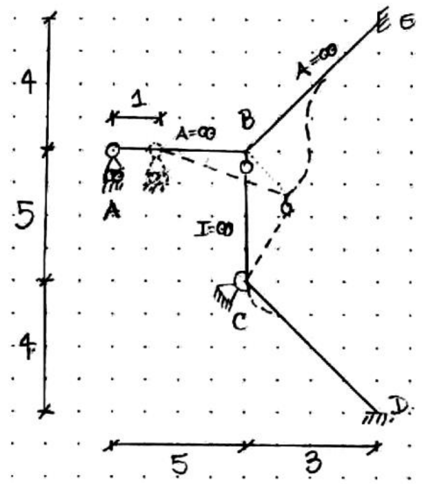
Q – D.



q – d

PREPARADOR: REDESCAR I. VIREL R.

∘ Modo I.



Como AB tiene $A=\infty$.

$\delta^h_A = \delta^h_B = 1$

$\varphi_{BE} = \dfrac{\delta^h_B}{dv_{B\text{-polo}}} = 1/4$

$\varphi_{BC} = -1/5$

$|\Delta L^{BC}| = \delta^v_B = \varphi_{BE} \times dh_{B\text{-polo}}$

$|\Delta L^{BC}| = 1/4 \times 3 = 3/4 \Rightarrow \Delta L^{BC} = -3/4$

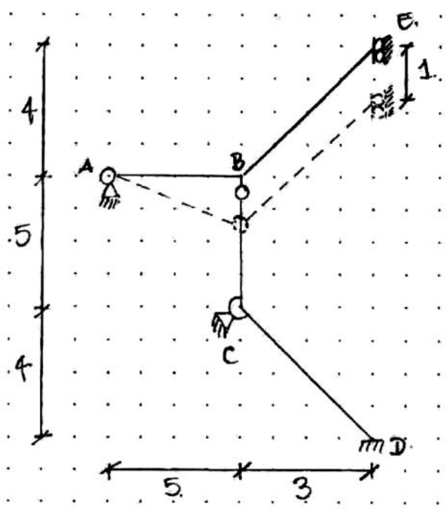
$\varphi_{AB} = \dfrac{3/4}{5} = -3/20$

$d_{1i} \begin{cases} 0 - 1/4 \\ -3/20 - 1/4 \\ -3/4 \\ 0 \\ -1/5 \\ 0 \end{cases}$

$\theta_A = \theta_B\uparrow = -3/20$

$\theta_B\downarrow = \theta_C = -1/5 \quad \theta_D = \theta_E = 0$

∘ Modo II.



$\Delta L^{BC} = -1$

$\varphi_{AB} = -1/5$

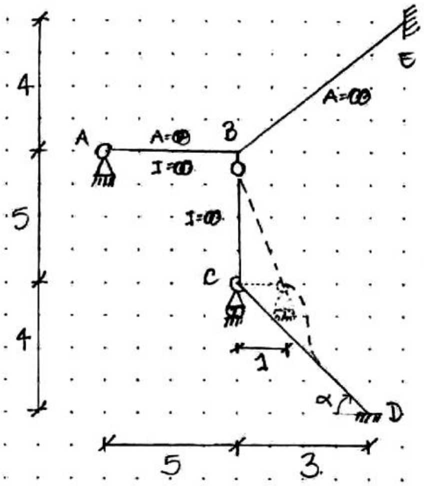
$\theta_A = \theta_B\uparrow = -1/5$

$\theta_B\downarrow = \theta_E = \theta_C = \theta_D = 0$

$d_{2i} \begin{cases} 0 \\ -1/5 \\ -1 \\ 0 \\ 0 \\ 0 \end{cases}$

PREPARADOR: REDESCAR J. VIREL 2

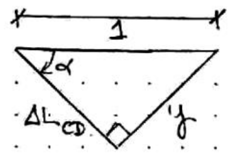
○ Modo III.



$\varphi_{BC} = 1/5$

$\theta_{B} = \theta_{C} = 1/5$

Para hallar el ΔL^{CD}:

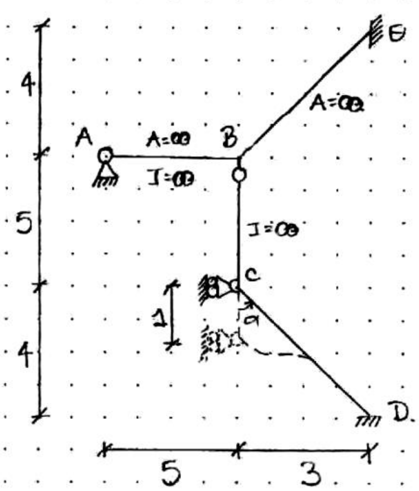


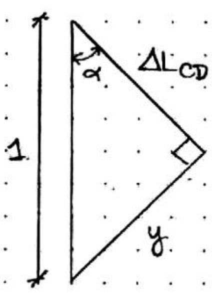
$\cos\alpha = 3/5 = -\Delta L^{CD}$

$\operatorname{sen}\alpha = 4/5 = y \Rightarrow \varphi_{CD} = \frac{-4/5}{5} = -4/25$

$$d_{3i} \begin{cases} 0 \\ 0 \\ 0 \\ 0-(-4/25) \\ 1/5-(-4/25) \\ -3/5 \end{cases}$$

○ Modo IV.





$\cos\alpha = 4/5 = |\Delta L_{CD}| \Rightarrow \Delta L_{CD} = -4/5$

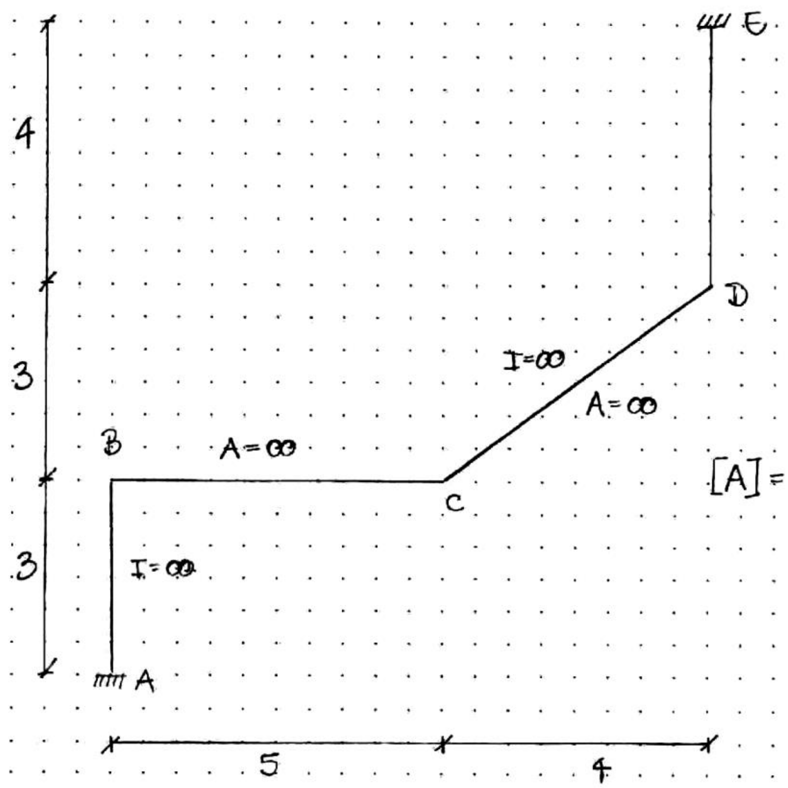
$\operatorname{sen}\alpha = 3/5 = y \Rightarrow \varphi_{CD} = \frac{3/5}{5} = 3/25$

$$d_{4i} \begin{cases} 0 \\ 0 \\ 1 \\ 0-(3/25) \\ 0-(3/25) \\ -4/5 \end{cases}$$

El Modo 5 corresponde únicamente a la rotación en "E", por lo que solo se afectará el término d_{51} cuyo valor será 1.

PREPARADOR: REDESCAR J YIREZ ?

9)



$$[A] = \begin{bmatrix} 1 & 0 & 0 \\ 1/5 & -9/20 & 1/4 \\ 1/5 & -1/5 & 0 \\ 0 & -3/16 & 3/16 \\ 0 & -7/16 & 7/16 \\ 0 & 0 & -1 \end{bmatrix}$$

$$I.G. = I.G.^{S/R} - \text{RESTRICCIONES} \quad (I)$$

$$I.G.^{S/R} = \underset{(B)}{3} + \underset{(C)}{3} + \underset{(D)}{3} = 9$$

$$\text{RESTRICCIONES} = \underset{(A-B)}{2} + \underset{(B-C)}{1} + \underset{(C-D)}{3}$$

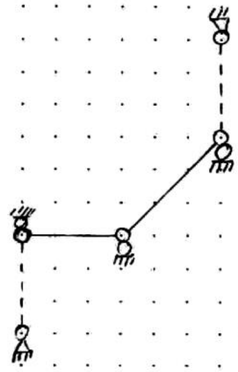
$$\boxed{I.G. = 9 - 6 = 3}$$

PREPARADOR: REDESCAB. J. YIREL R.

$$I.G. = G.D. + R.I. \quad (II)$$

- Para hallar los Grados de Desplazabilidad

- IMAGEN CINEMÁTICA

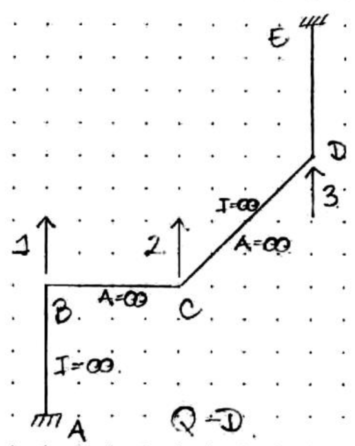


$GD = 3$ y $R.I. = 0$

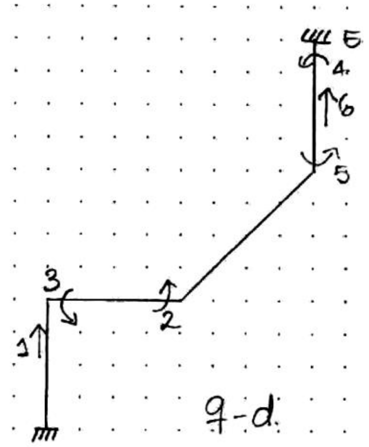
Luego del contenido estudiado hasta el momento surge la pregunta: ¿Por Qué la imagen cinemática no admite un desplazamiento horizontal en la junta B?

$$\boxed{I.G. = 3 + 0 = 3}$$

- SISTEMA GLOBAL

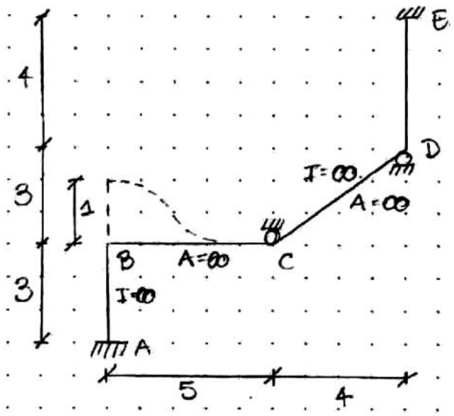


- SISTEMA LOCAL



PREPARADOR: REDESCAR J. VIRLA R.

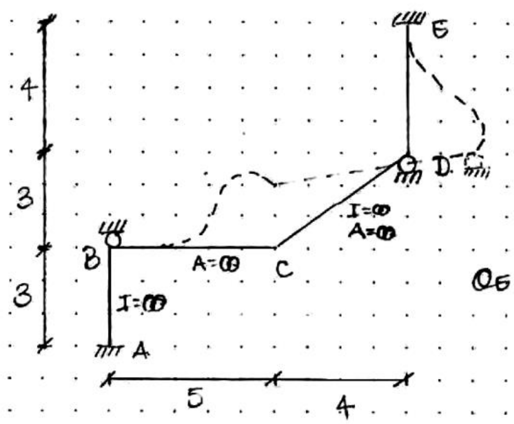
° Modo I



$\varphi_{BC} = -1/5$

$$d_{1i} \begin{cases} 1 \\ 1/5 \\ 1/5 \\ 0 \\ 0 \\ 0 \end{cases}$$

° Modo II



$\delta_C^v = 1 \Rightarrow \varphi_{BC} = 1/5$

$\varphi_{CD} = -1/4$

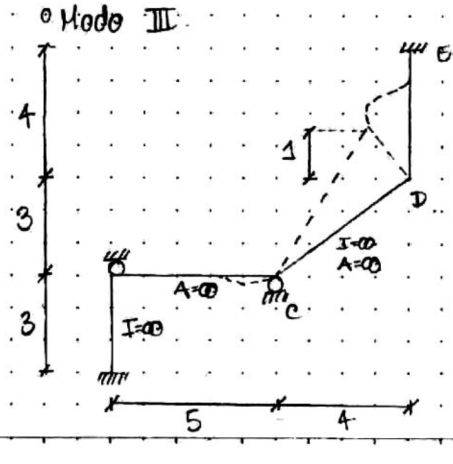
$\varphi_{DE} = \frac{3/4}{4} = 3/16$

$\theta_E = \theta_A = \theta_B = 0$

$\theta_C = \theta_D = -1/4$

$$d_{2i} \begin{cases} 0 \\ -1/4 - 1/5 \\ 0 - 1/5 \\ -3/16 \\ -1/4 - 3/16 \\ 0 \end{cases}$$

° Modo III



$\delta_D^v = 1 = |\Delta L_{DE}|$

$\varphi_{CD} = 1/4$

$\delta_D^h = 1/4 \times 3 = 3/4$

$\varphi_{DE} = -3/16$

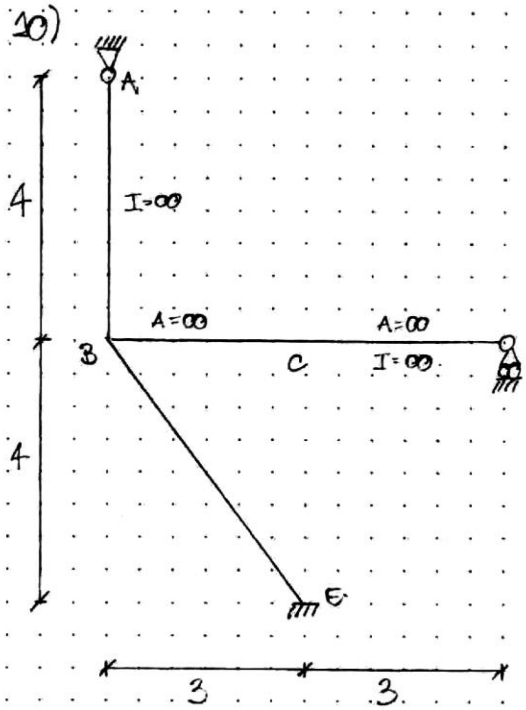
$\theta_E = \theta_A = \theta_B = 0$

$\theta_C = \theta_D = 1/4$

$$d_{3i} \begin{cases} 0 \\ 1/4 \\ 0 \\ 3/16 \\ 1/4 + 3/16 \\ -1 \end{cases}$$

Preparador Redebar J. Virla R.

10)



$$[A]=\begin{bmatrix} -1 & 0 & 0 \\ 1/3 & -2/3 & 0 \\ 1/3 & -1/3 & 1/4 \\ 3/25 & 0 & 4/25 \\ 3/25 & 0 & 41/100 \\ 4/5 & 0 & -3/5 \end{bmatrix}$$

$I.G. = I.G^{S/R} - \text{RESTRICCIONES} \quad (I)$

$I.G^{S/R} = \underset{(A)}{1} + \underset{(B)}{3} + \underset{(C)}{3} + \underset{(D)}{2} = 9$

RESTRICCIONES: $\underset{(A-B)}{2} + \underset{(B-C)}{1} + \underset{(C-D)}{3} = 6$

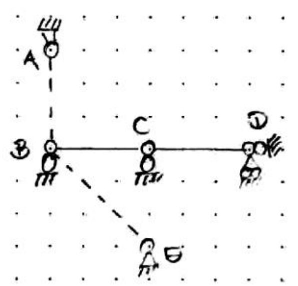
$$\boxed{I.G. = 9 - 6 = 3}$$

PREPARADOR: REDESCAR J. VIREL R.

$$I.G. = G.D + R.I$$

-Para hallar los Grados de Desplazabilidad:

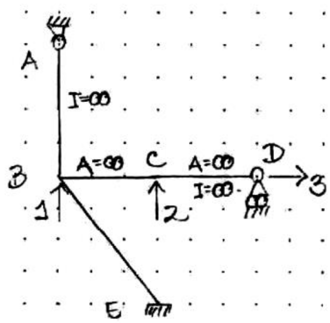
∘ IMAGEN CINEMÁTICA:



$GD = 3$ y $R.I. = 0$

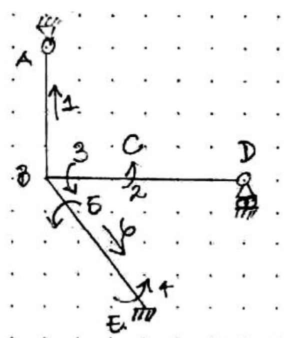
$$\boxed{I.G. = 3 + 0 = 3}$$

∘ SISTEMA GLOBAL



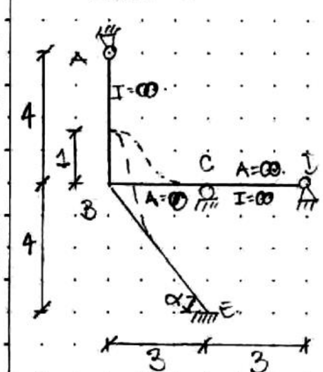
Q-D.

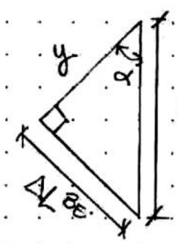
∘ SISTEMA LOCAL



q-d.

∘ Modo I





$\varphi_{BC} = -1/3$

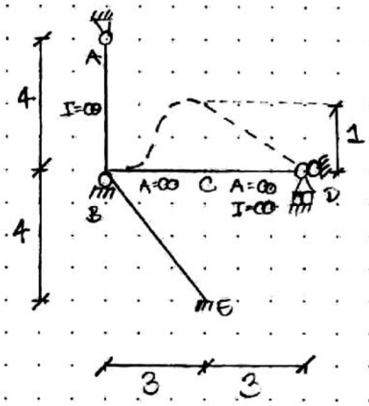
$\operatorname{sen}\alpha = 4/5 = \Delta L_{BE}$

$\cos\alpha = 3/5 = y \Rightarrow \varphi_{BE} = \frac{-3/5}{5} = -3/25$

$$d_{1i} \begin{cases} -1 \\ 1/3 \\ 1/3 \\ 3/25 \\ 3/25 \\ 4/5 \end{cases}$$

PREPARADOR REDESCAR J. VIREL R

° Modo II.

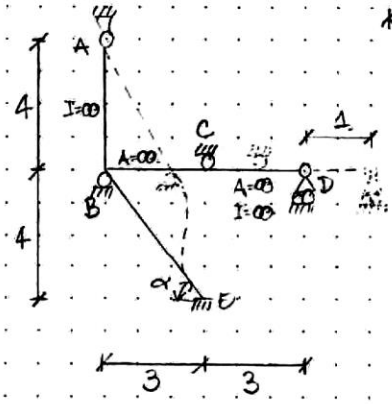


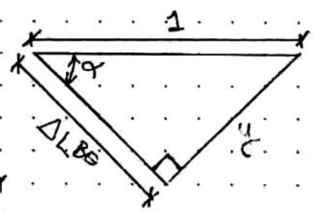
$\varphi_{BC} = 1/3$

$\varphi_{CD} = -1/3$

$$d_{i2} \left\{ \begin{array}{l} 0 \\ -1/3 - 1/3 \\ -1/3 \\ 0 \\ 0 \\ 0 \end{array} \right.$$

° Modo III.





$\cos\alpha = 3/5 = |\Delta L^{BE}|$

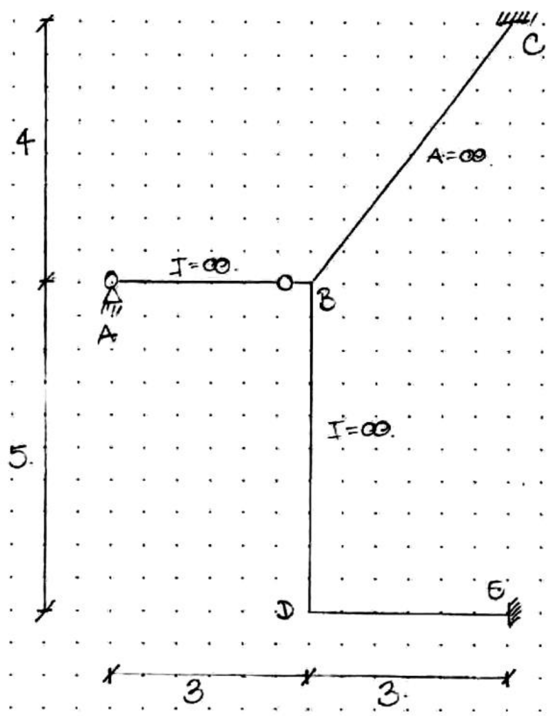
$\operatorname{sen}\alpha = 4/5 = y \Rightarrow \varphi_{BE} = -\frac{4/5}{5} = -4/25$

$\varphi_{AB} = 1/4$

$$d_{i3} \left\{ \begin{array}{l} 0 \\ 0 \\ 1/4 \\ 4/25 \\ 1/4 + 4/25 \\ -3/5 \end{array} \right.$$

PREPARADOR: REDIBUJAR J. VIREZ R.

11)



$$[A] = \begin{bmatrix} 1 & 0 & 0 \\ -1/4 & 0 & 0 \\ -9/20 & 0 & 1/4 \\ -3/4 & -1 & 0 \\ 0 & 1/3 & 0 \\ -1/5 & 1/3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$I.G. = I.G.^{S/R} - \text{RESTRICCIONES}$$

$$I.G.^{S/R} = \underset{(A)}{1} + \underset{(B)}{4} + \underset{(D)}{3} = 8$$

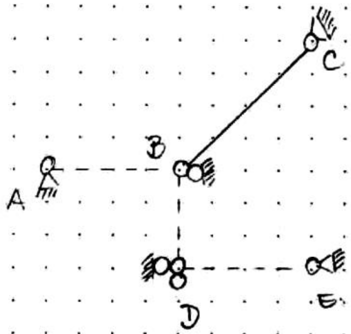
$$\text{RESTRICCIONES} = \underset{(A-B)}{2} + \underset{(B-C)}{1} + \underset{(B-D)}{2} = 5$$

$$\boxed{I.G. = 8 - 5 = 3}$$

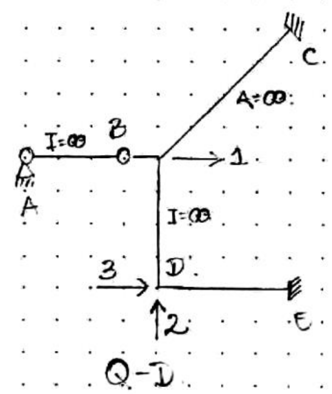
PREPARADOR: REDEBAR : VIREL R.

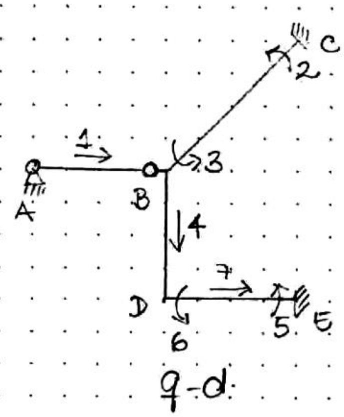
$$I.G. = G.D. + R.I. \quad (II).$$

- Para hallar los Grados de Desplazabilidad:

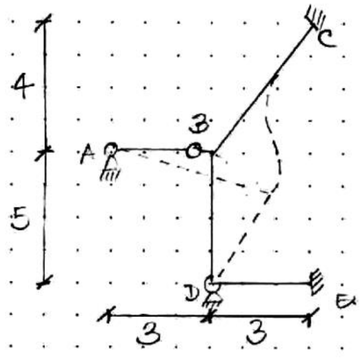


$G.D. = 3$ y $R.I. = 0$





- Modo I:



$\varphi_{BC} = 1/4$

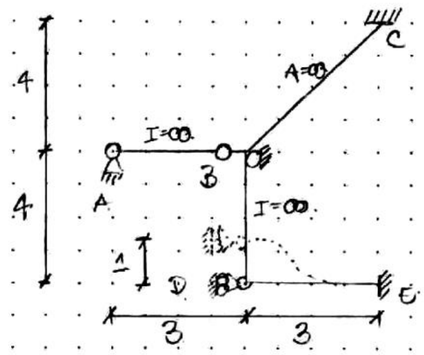
$\varphi_{BD} = -1/5$

$\delta_B^V = 1/4 \times 3 = 3/4 \Rightarrow \varphi_{BA} = \frac{-3/4}{3} = -1/4$

$$d_{1i} \begin{cases} 1 \\ -1/4 \\ -1/5 - 1/4 \\ -3/4 \\ 0 \\ -1/5 \\ 0 \end{cases}$$

Preparador: Rederpar J. Virel R

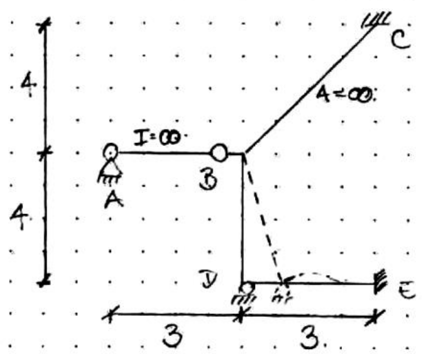
○ Modo II



$\psi_{DE} = -1/3$.

$$d_{2i} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 1/3 \\ 1/3 \\ 0 \end{Bmatrix}$$

○ Modo III

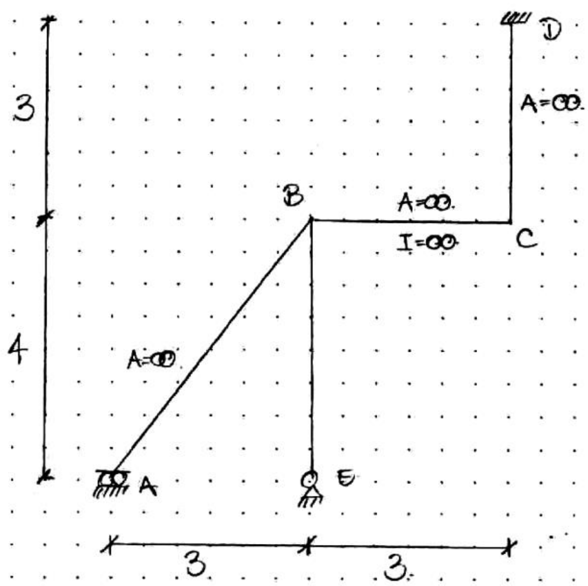


$\psi_{BD} = 1/4$

$$d_{3i} = \begin{Bmatrix} 0 \\ 0 \\ 1/4 \\ 0 \\ 0 \\ 0 \\ -1 \end{Bmatrix}$$

Preparador: Redescar J. Virel P.

12)



$$[A] = \begin{bmatrix} -2/3 & 0 & 0 \\ -1/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ -1/3 & -1/3 & 0 \\ -1/3 & 1/4 & 0 \\ 0 & 1/4 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$I.G. = I\,G^{s/R} - \text{RESTRICCIONES} \quad (I)$

$I.G^{s/R} = \underset{(A)}{1} + \underset{(B)}{3} + \underset{(C)}{3} + \underset{(E)}{1} = 8$

$\text{RESTRICCIONES} = \underset{(A\text{-}B)}{1} + \underset{(B\text{-}C)}{3} + \underset{(C\text{-}D)}{1} = 5$

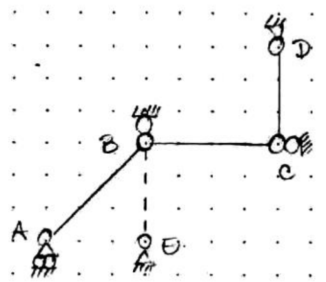
$\boxed{I.G. = 8 - 5 = 3}$

Preparador: Redescar J. Virel R.

I.G. = G.D. + R.I. (II)

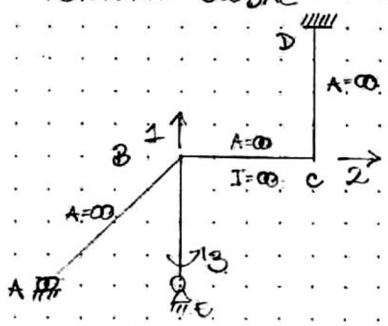
- Para hallar los Grados de Desplazabilidad:

° IMAGEN CINEMÁTICA:



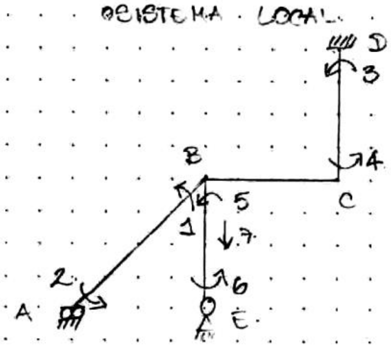
$G.D. = 2$ y $R.I. = 1$ (E)

° SISTEMA GLOBAL



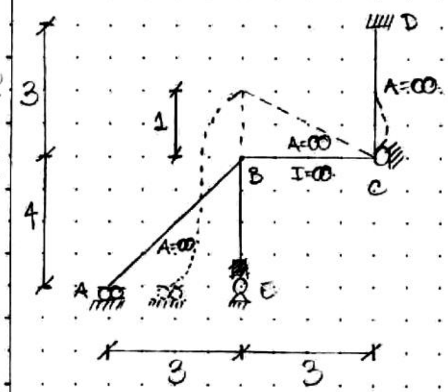
Q-D

° SISTEMA LOCAL



q-d

° Modo I



$\varphi_{BC} = -1/3$

$\varphi_{AB} = 1/3$

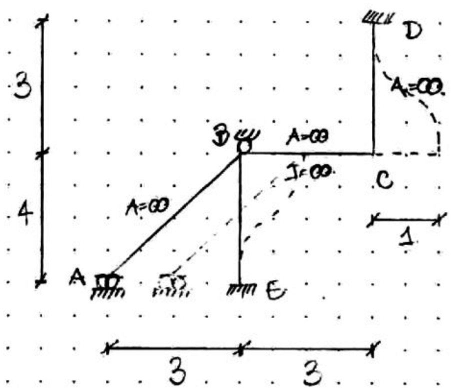
$\Delta L^{BE} = 1$

$\theta_B = \theta_C = -1/3$

$$d_{1i} \left\{ \begin{array}{c} -1/3 - 1/3 \\ -1/3 \\ 0 \\ -1/3 \\ -1/3 \\ 0 \\ 1 \end{array} \right.$$

Preparador: Redesar J. Virel R.

o Modo II.

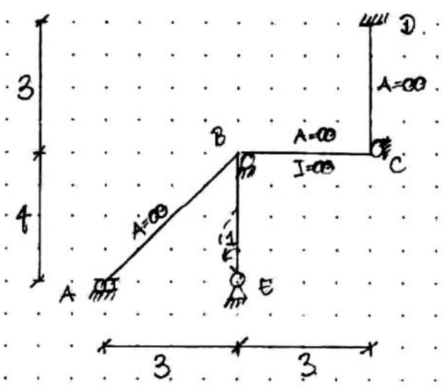


$\psi_{CD} = 1/3$

$\psi_{BE} = -1/4$

$$d_{2i} \begin{Bmatrix} 0 \\ 0 \\ -1/3 \\ -1/3 \\ 1/4 \\ 1/4 \\ 0 \end{Bmatrix}$$

o Modo III.

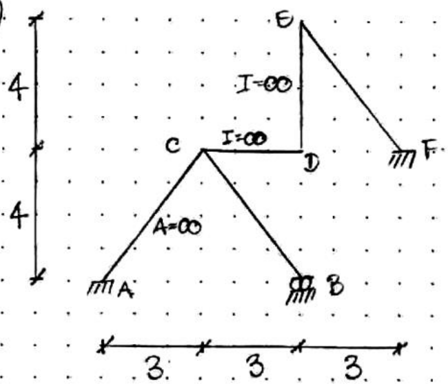


$$d_{3i} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{Bmatrix}$$

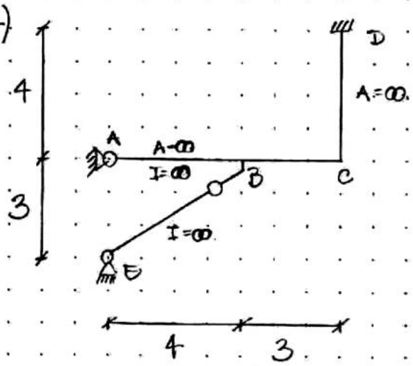
PREPARADOR: REDESCAR J. VIREL R

○ EJERCICIOS PROPUESTOS.

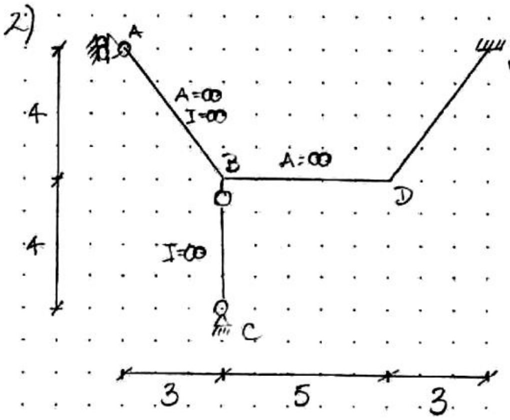
1)



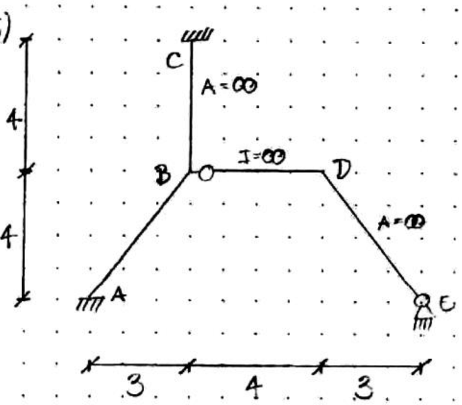
4)



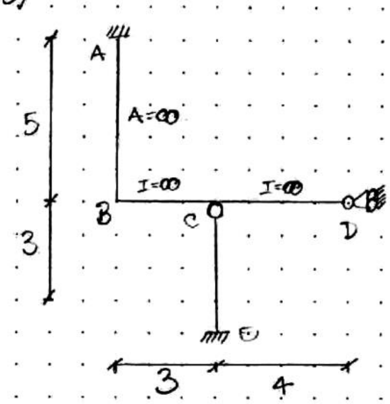
2)



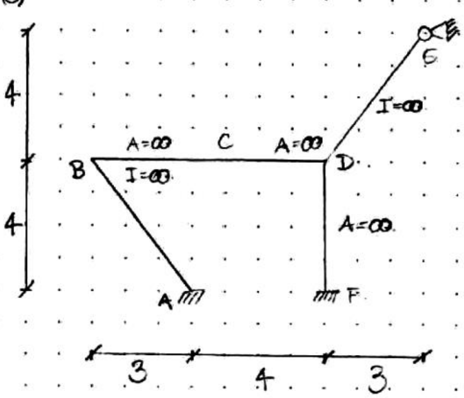
5)



3)

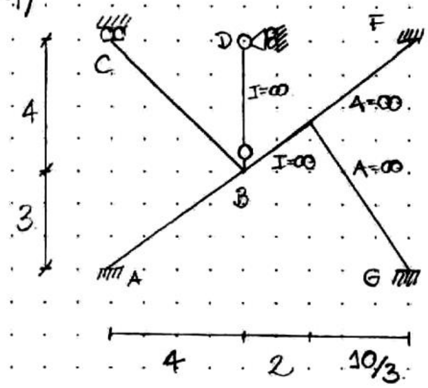


6)



PREPARADOR: REDESCAR J VIREL R.

7)



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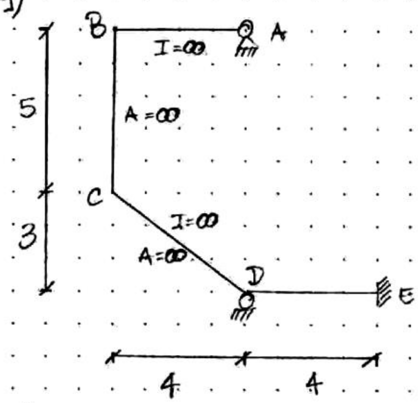

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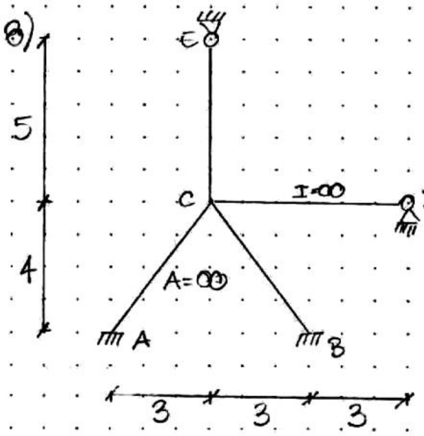

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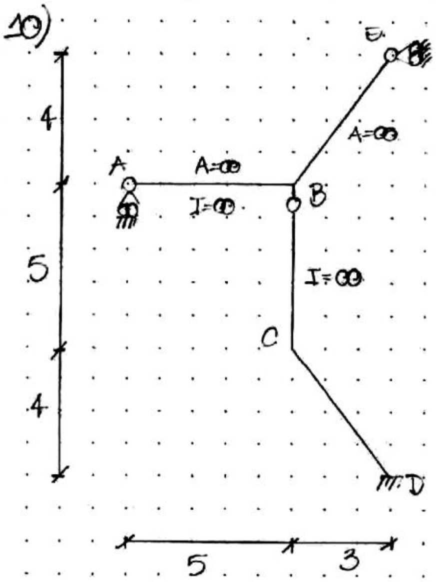

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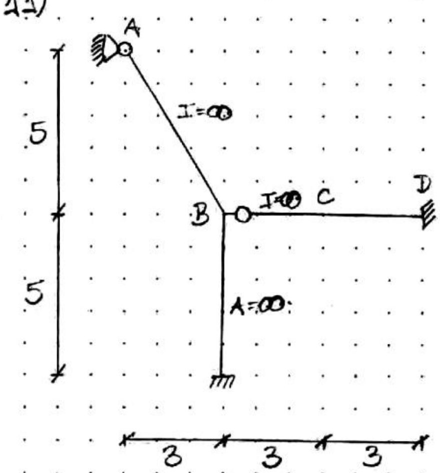

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