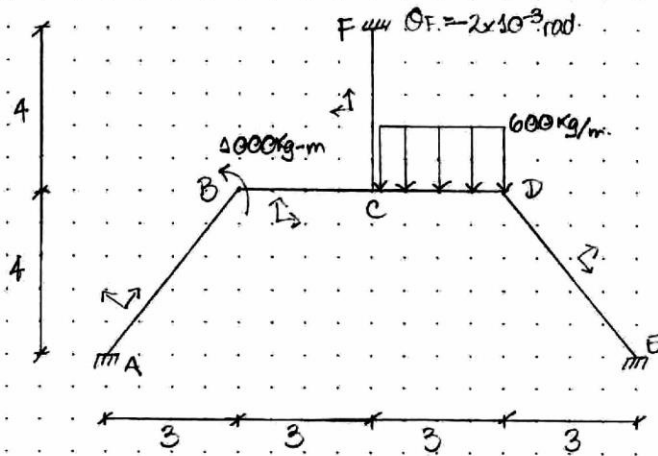
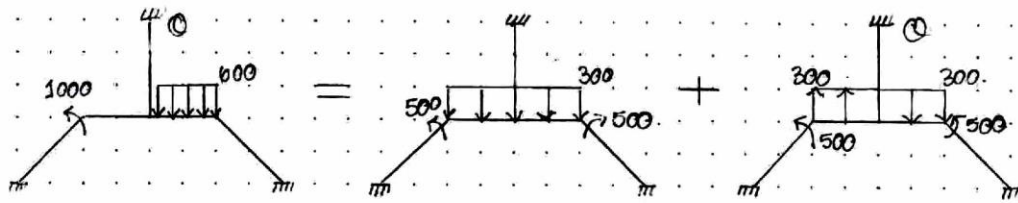


7) Determine los Diagramas de Momento:



° ESTRUCTURA SIMÉTRICA SIN DISPOSICIÓN PARTICULAR DE LAS CARGAS

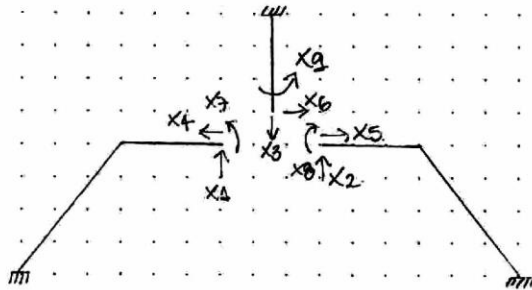
- El problema se debe sub-dividir en un caso simétrico y uno antisimétrico, que mediante el principio de superposición, permita darle solución al problema original.



PREGUNTAS: REDESARROLLAR = 1/18/12 R

ESTRUCTURA SIMÉTRICA CON CARGAS SIMÉTRICAS.

- Se plantea el corte de la estructura.

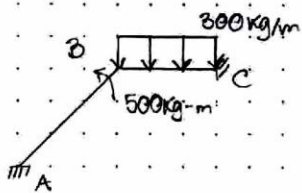


$$x_1 = x_2 \quad \Rightarrow \quad x_3 = x_4 + x_2$$

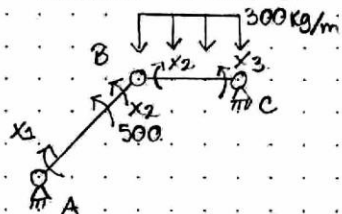
$$x_4 = x_5 \quad \Rightarrow \quad x_6 = 0$$

$$x_7 = x_8 \quad \Rightarrow \quad x_9 = 0$$

El sistema equivale a:



- Sistema Primario:

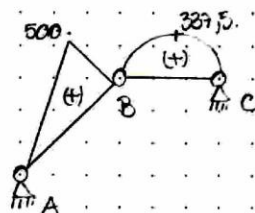
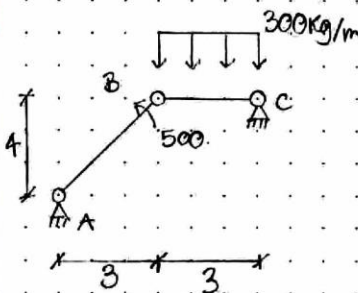


Ec. de Compatibilidad

$$D_1 = D_2 = D_3 = 0$$

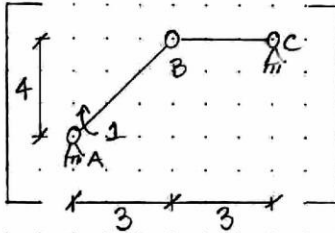
El trabajo realizado por las fuerzas x_1, x_2 y x_3 es cero.

- Sistema Cero

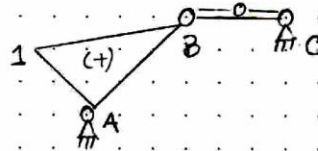


PREPARADOR: RODRIGUEZ J. V. IREAZ R.

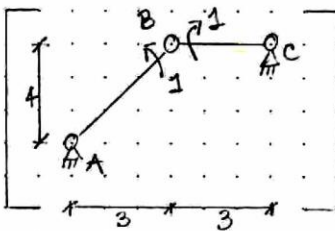
- Sistema Uno.



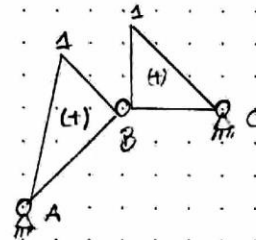
• X1



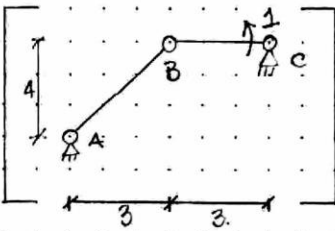
- Sistema Dos.



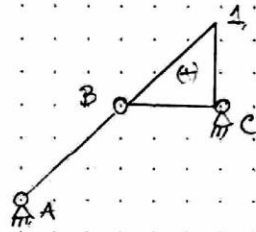
• X2



- Sistema Tres.



• X3



$$D_1 = D_{10} + d_{11} \cdot X_1 + d_{12} \cdot X_2 + d_{13} \cdot X_3$$

$$D_2 = D_{20} + d_{21} \cdot X_1 + d_{22} \cdot X_2 + d_{23} \cdot X_3$$

$$D_3 = D_{30} + d_{31} \cdot X_1 + d_{32} \cdot X_2 + d_{33} \cdot X_3$$

← PRINCIPIO DE SUPERPOSICIÓN

$$D_{10} = \frac{1 \times 500 \times 5}{3 \times 10} = \frac{2500}{3 \times 10}$$

$$d_{13} = 0$$

$$d_{22} = \frac{1 \times 1 \times 5}{3 \times 10} + \frac{1 \times 1 \times 3}{3 \times 10} = \frac{8}{3 \times 10}$$

$$d_{11} = \frac{1 \times 1 \times 5}{3 \times 10} = \frac{5}{3 \times 10}$$

$$D_{20} = \frac{500 \times 1 \times 5}{3 \times 10} + \frac{337,5 \times 1 \times 3}{3 \times 10}$$

$$d_{23} = \frac{1 \times 1 \times 3}{6 \times 10} = \frac{1}{2 \times 10}$$

$$d_{12} = \frac{1 \times 1 \times 5}{6 \times 10} = \frac{5}{6 \times 10}$$

$$D_{30} = \frac{7025}{6 \times 10}$$

$$D_{30} = \frac{337,5 \times 1 \times 3}{3 \times 10} = \frac{337,5}{10}$$

PREPARADOR: PEDRO CAR J. VIREL R.

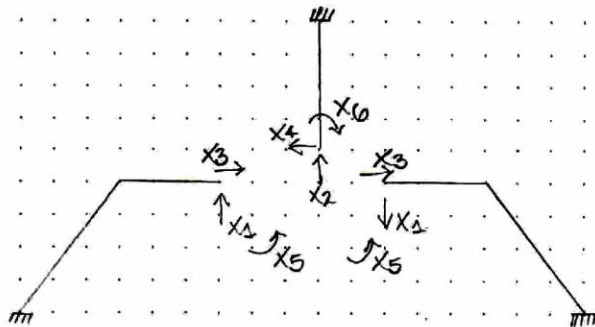
$$D_{33} = \frac{4 \times 4 \times 3}{3EI_0} = \frac{4}{EI_0}$$

$$\begin{cases} 0 = \frac{2500}{3EI_0} + \frac{5}{3EI_0} \cdot X_1 + \frac{5}{6EI_0} \cdot X_2 & \text{(I)} \\ 0 = \frac{7025}{6EI_0} + \frac{5}{6EI_0} \cdot X_1 + \frac{8}{3EI_0} \cdot X_2 + \frac{1}{2EI_0} \cdot X_3 & \text{(II)} \\ 0 = \frac{337,5}{EI_0} + \frac{1}{2EI_0} \cdot X_2 + \frac{X_3}{EI_0} & \text{(III)} \end{cases}$$

$$X_1 = -354 \text{ Kg-m} ; X_2 = -293 \text{ Kg-m} ; X_3 = -191 \text{ Kg-m}$$

° ESTRUCTURA SIMÉTRICA CON CARGAS ANTISIMÉTRICAS.

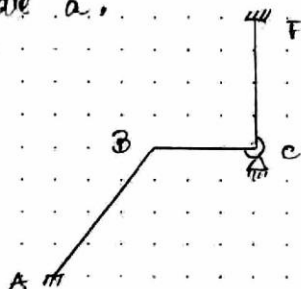
- Se plantea el corte de la estructura



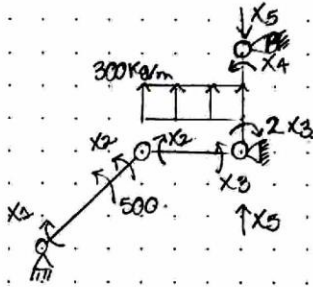
$$\begin{aligned} X_1 \neq & \Rightarrow X_2 = 0 \\ X_3 \neq & \Rightarrow 2X_3 = X_4 \\ X_5 \neq & \Rightarrow 2X_5 = X_6 \end{aligned}$$

Para considerar la influencia del movimiento de apoyo en el eje de simetría, el problema debe ser resuelto considerando el eje.

- El sistema equivale a:



- Sistema Primario



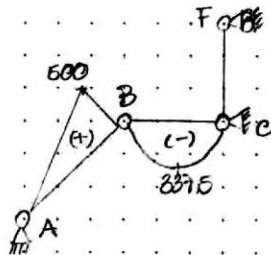
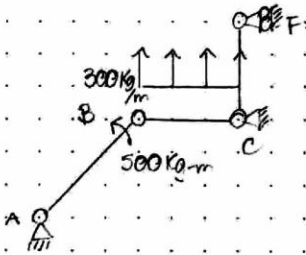
Excepciones de Compatibilidad

$$D_1 = D_2 = D_3 = D_5 = 0$$

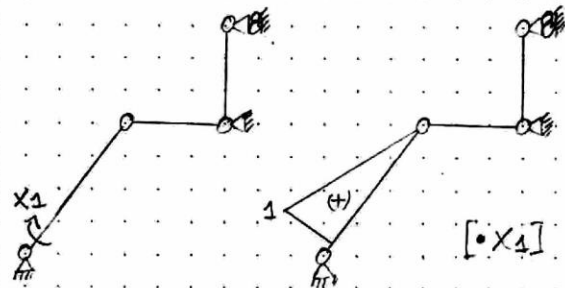
El trabajo realizado por las fuerzas X_1, X_2, X_3 y X_5 es cero

$$D_4 = -2 \times 10^{-3} \text{ rad}$$

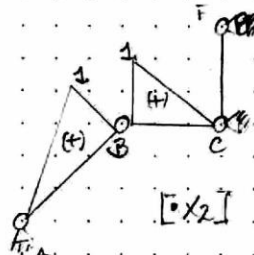
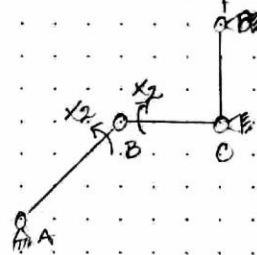
- Sistema Cero



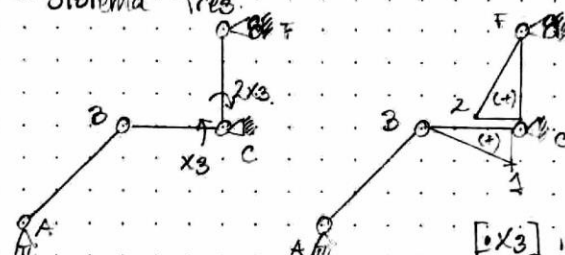
- Sistema Uno



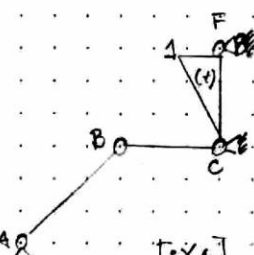
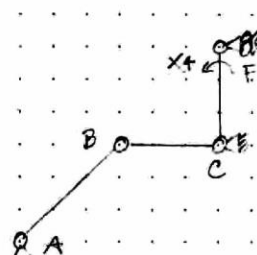
- Sistema Dos



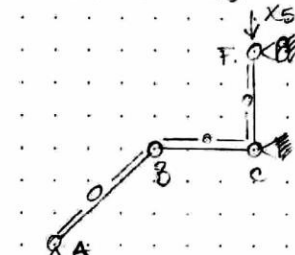
- Sistema Tres



- Sistema Cuatro



- Sistema Cinco



PREPARADOR: REDDIBAGAZ J. VIREL R.

Proyecto TEMA 3 - MÉTODO DE LAS FUERZAS.

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$$D_{10} = \frac{1 \times 500 \times 5}{3EJ_0} = \frac{2500}{3EJ_0}$$

$$d_{11} = \frac{1 \times 1 \times 5}{3EJ_0} = \frac{5}{3EJ_0}; \quad d_{12} = \frac{-1 \times 1 \times 5}{6EJ_0} = \frac{5}{6EJ_0}; \quad d_{13} = 0; \quad d_{14} = 0; \quad d_{15} = 0.$$

$$D_{20} = \frac{1 \times 500 \times 5}{3EJ_0} + \frac{1 \times (-337,5) \times 3}{3EJ_0} = \frac{2975}{6EJ_0}$$

$$d_{22} = \frac{1 \times 1 \times 5}{3EJ_0} + \frac{1 \times 1 \times 3}{3EJ_0} = \frac{8}{3EJ_0}; \quad d_{23} = \frac{1 \times 1 \times 3}{6EJ_0} = \frac{1}{2EJ_0}; \quad d_{24} = 0; \quad d_{25} = 0.$$

$$D_{30} = \frac{(-337,5) \times (1) \times 3}{3EJ_0} = \frac{-337,5}{EJ_0}$$

$$d_{33} = \frac{1 \times 1 \times 3}{3EJ_0} + \frac{2 \times 2 \times 4}{3EJ_0} = \frac{19}{3EJ_0}; \quad d_{34} = \frac{2 \times 1 \times 4}{6EJ_0} = \frac{4}{3EJ_0}; \quad d_{35} = 0.$$

$$D_{40} = 0; \quad D_{50} = 0.$$

$$d_{44} = \frac{1 \times 1 \times 4}{3EJ_0} = \frac{4}{3EJ_0}; \quad d_{45} = 0; \quad d_{55} = 0.$$

$$0 = \frac{2500}{3EJ_0} + \frac{5}{3EJ_0} \cdot X_1 + \frac{5}{6EJ_0} \cdot X_2$$

$$X_1 = -429 \text{ Kg-m}$$

$$0 = \frac{2975}{6EJ_0} + \frac{5}{6EJ_0} \cdot X_1 + \frac{8}{3EJ_0} \cdot X_2 + \frac{X_3}{2EJ_0}$$

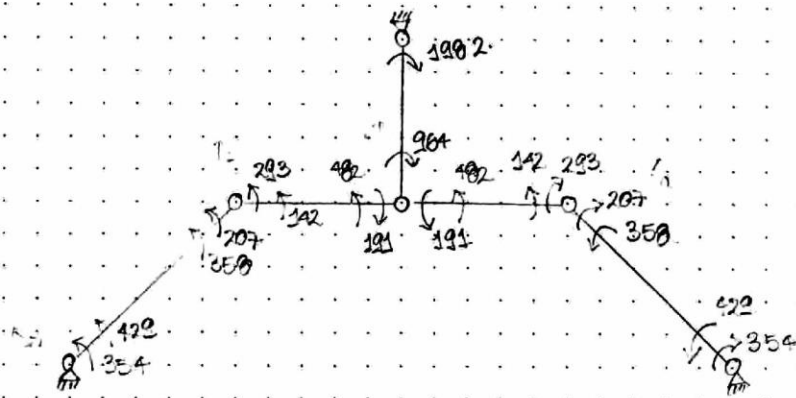
$$X_2 = -142 \text{ Kg-m}$$

$$X_3 = 482 \text{ Kg-m}$$

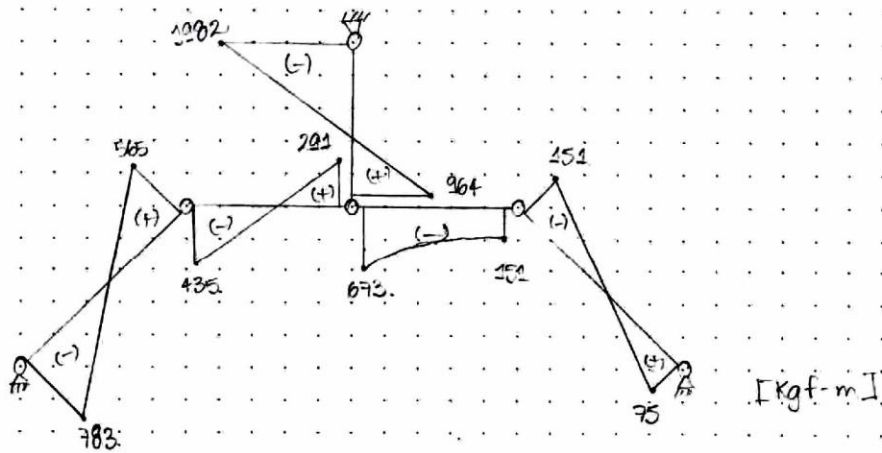
$$0 = \frac{-337,5}{EJ_0} + \frac{X_2}{2EJ_0} + \frac{19}{3EJ_0} \cdot X_3 + \frac{4}{3EJ_0} \cdot X_4$$

$$X_4 = -1982 \text{ Kg-m}$$

$$\frac{-2000}{EJ_0} = \frac{4}{3EJ_0} \cdot X_3 + \frac{4}{3EJ_0} \cdot X_4$$

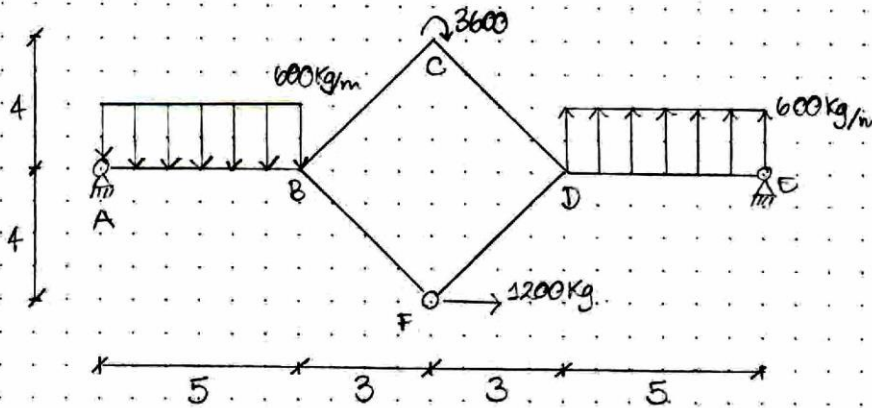


-Diagrama de Momentos-



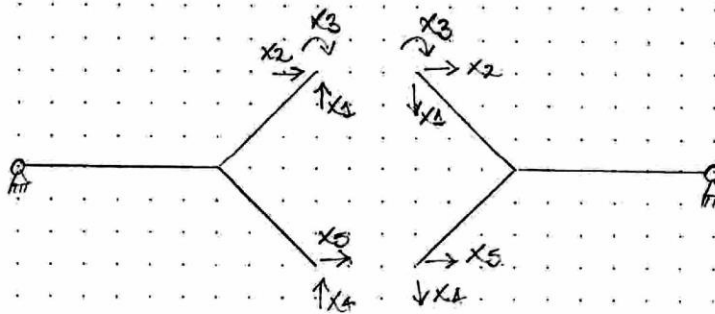
PREPARADOR: RESERVA 1/12 EL R

B) Grafique los Diagramas de Momento



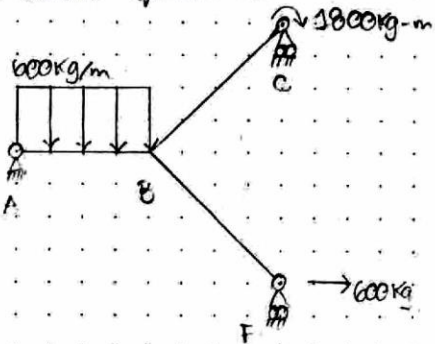
• ESTRUCTURA SIMÉTRICA CON CARGAS ANTISIMÉTRICAS

- Se plantea el corte de la estructura:



$$\begin{aligned} X_1 &= 0 \\ X_2 &= 0 \\ 2X_3 &= 3600 \text{ Kg-m} \\ X_4 &= 0 \\ 2X_5 &= 1200 \text{ Kg} \end{aligned}$$

El sistema equivale a:

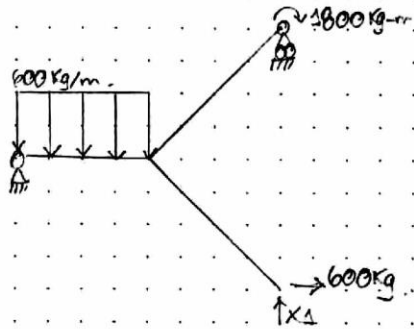


$$GH = 1$$

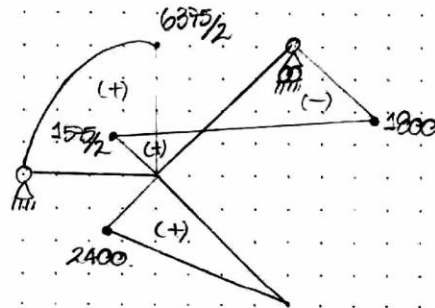
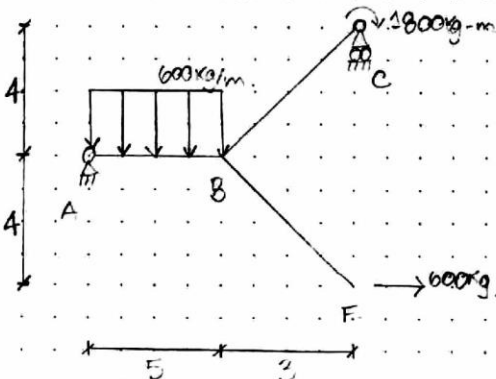
PREPARADOR: ROBERTO J. VIREL R.

MÉTODO DE LAS FUERZAS

Sistema Primario



Sistema C.a.o.

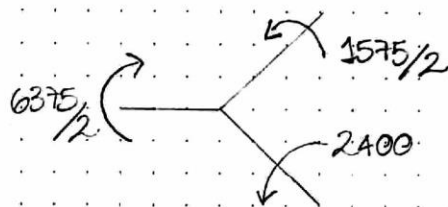


$$\sum M_A^S = 0 \Rightarrow C_v \times 8 - 1800 + 600 \times 4 - 600 \times 5 \times \frac{5}{2} = 0 \Rightarrow C_v = \frac{1725}{2}$$

$$\sum M_B^S = 0 \Rightarrow -M_{BC} - 1800 + C_v \times 3 = 0 \Rightarrow M_{BC} = 1575/2$$

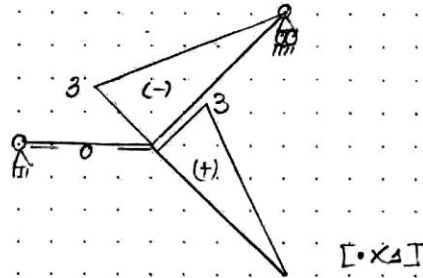
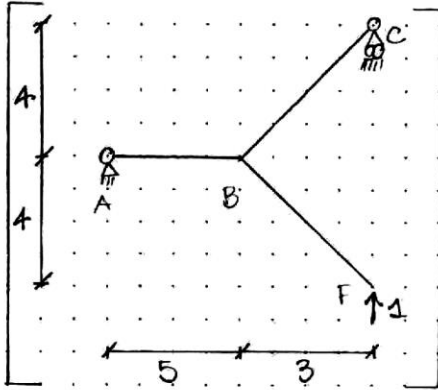
$$\sum M_B^S = 0 \Rightarrow -M_{BF} + 600 \times 4 = 0 \Rightarrow M_{BF} = 2400$$

Equilibrio junta B



PREPARADOR: RIVEROS R. VILSA R.

Sistema Uno.



$$\sum \overset{+}{M}_A = 0 \Rightarrow C_v \times 8 + 0 = 0 \Rightarrow C_v = -1$$

Por inspección $A_v = 0 \Rightarrow M_{BA} = 0$

$$\sum \overset{+}{M}_{BC} = 0 \Rightarrow C_v \times 3 - M_{BC} = 0 \Rightarrow M_{BC} = -3$$

$$D_1 = D_{10} + d_{11} \cdot X_1$$

$$D_{10} = -3 \times \frac{1575 \times 5}{2 \times 3E10} + (-1000) \times \frac{(-3) \times 5}{6E10} + \frac{3 \times 2400 \times 5}{3E10} = \frac{25125}{2E10}$$

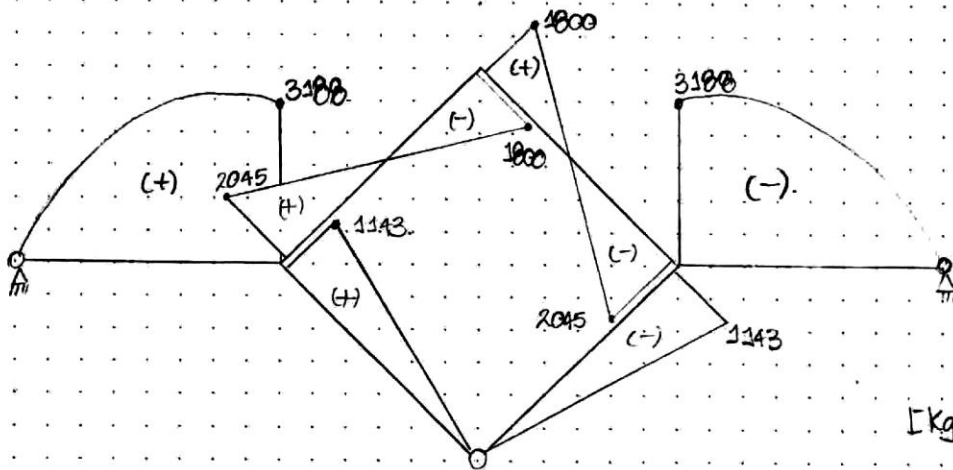
$$d_{11} = \frac{(-3) \times (-3) \times 5}{3E10} + \frac{3 \times 3 \times 5}{3E10} = \frac{30}{E10}$$

$$0 = \frac{25125}{2E10} + \frac{30}{E10} \cdot X_1 \Rightarrow X_1 = -419 \text{ Kg}$$

$$M_{BC} = M_{BC}^{(0)} + M_{BC}^{(1)} \cdot X_1 = 2045 \text{ Kg-m (v)}$$

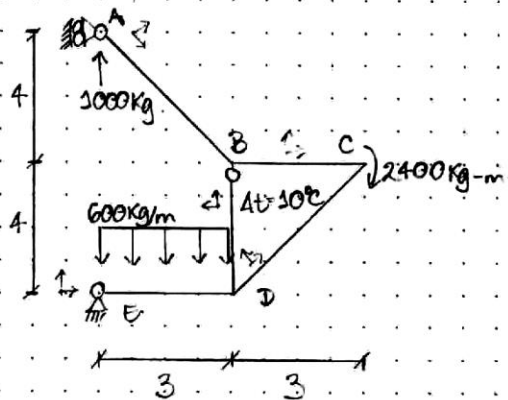
$$M_{BF} = M_{BF}^{(0)} + M_{BF}^{(1)} \cdot X_1 = 1143 \text{ Kg-m (v)}$$

ODIAGRAMAS FINALES.



PREPARADOR: PEDRO CAR J. VILCILL R.

1) Resuelva por el método de las fuerzas:

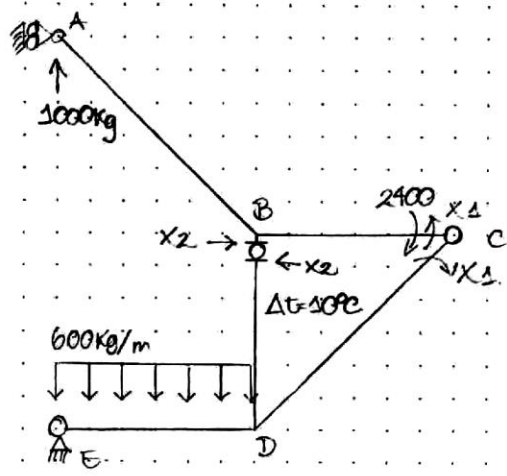


$\bar{V}_E = -3 \times 10^{-2} \text{ m}$
 $h = 0,80 \text{ m}$
 $EI_0 = 1 \times 10^6 \text{ kg-m}^2$

Por inspección, se puede interpretar que el sistema posee una hiperestaticidad interna de grado dos (2).

MÉTODO DE LAS FUERZAS

- Sistema Primario



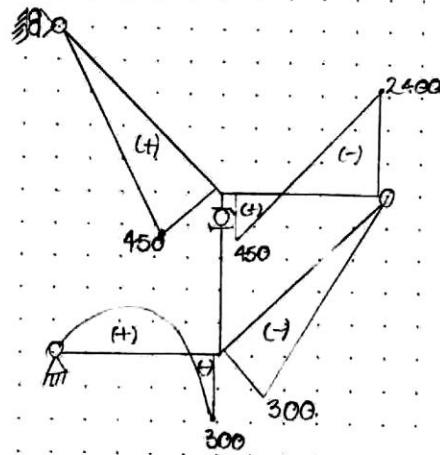
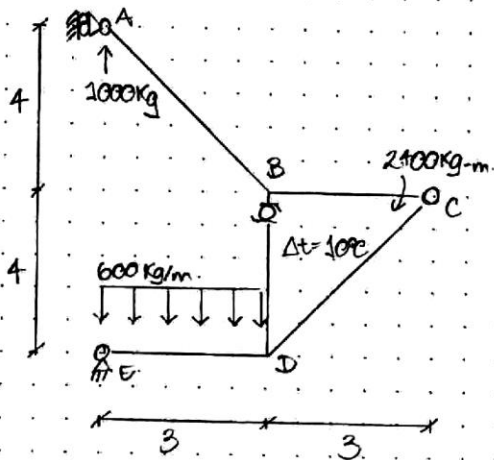
Ec. de Compatibilidad

$D_1 = 0, D_2 = 0$

El trabajo realizado por las fuerzas x_1 y x_2 es nulo

Problemas Resueltos I y II

- Sistema Cero



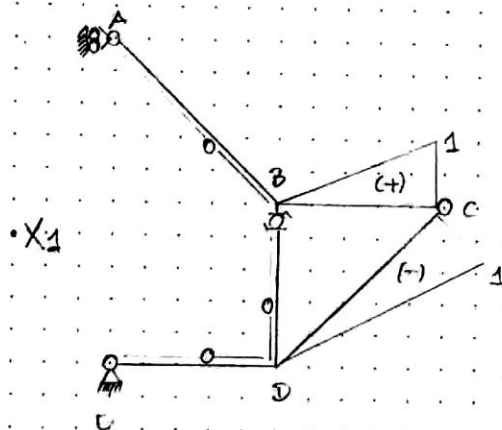
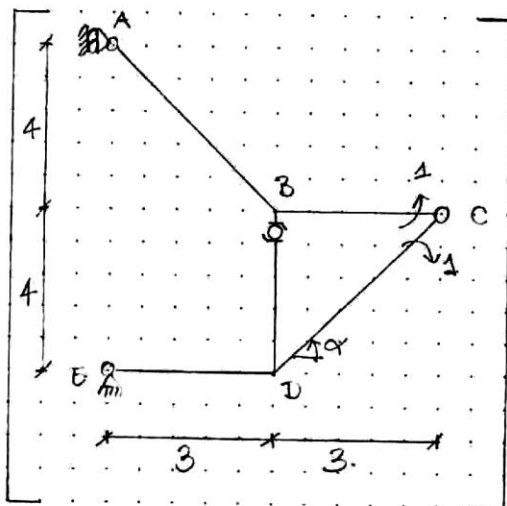
$$\sum M_E^{\uparrow} = 0 \quad -600 \times 3 \times \frac{3}{2} - 2400 - A_h \times 8 = 0 \Rightarrow A_h = -637,5$$

$$\sum H_B^{\rightarrow} = 0 \quad V_{BA} - 1000 \times 3 - A_h \times 4 = 0 \Rightarrow V_{BA} = 450$$

$$\uparrow \sum F_v = 0 \quad E_v - 600 \times 3 + 1000 = 0 \Rightarrow E_v = 800$$

$$\sum M_D^{\uparrow} = 0 \quad +M_{DE} + 600 \times 3 \times \frac{3}{2} - E_v \times 3 = 0 \Rightarrow M_{DE} = -300$$

- Sistema Uno

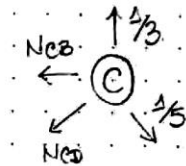


PREPARADOR: RODRIGUEZ C. VIVEL R.

$$\sum M_E^A = 0 \Rightarrow -8A_h = 0 \Rightarrow A_h = 0 \Rightarrow M_{BA} = 0$$

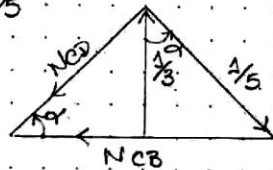
$$\uparrow \sum F_v = 0 \Rightarrow E_v = 0 \Rightarrow M_{DE} = 0$$

NODO "C"



$$M_{3(x)}_{DE} = \frac{1}{3}x \Rightarrow V_{(y)}_{DE} = \frac{1}{3}$$

$$M_{3(x)}_{DE} = -\frac{1}{5}x \Rightarrow V_{(y)}_{DE} = -\frac{1}{5}$$



sen $\alpha = 4/5$
cos $\alpha = 3/5$

$$\uparrow \sum F_v = 0 \Rightarrow -N_{ED} \sin \alpha + \frac{1}{3} - \frac{1}{5}x \cos \alpha = 0$$

$$N_{ED} = \frac{4}{15}$$

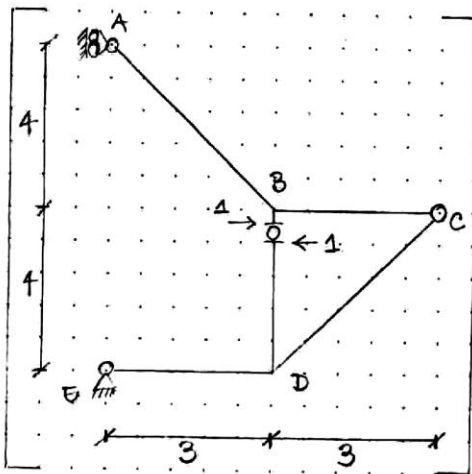
$$\rightarrow \sum F_h = 0 \Rightarrow -N_{CB} + \frac{1}{5}x \sin \alpha - N_{ED} \cos \alpha = 0$$

$$N_{CB} = 0$$

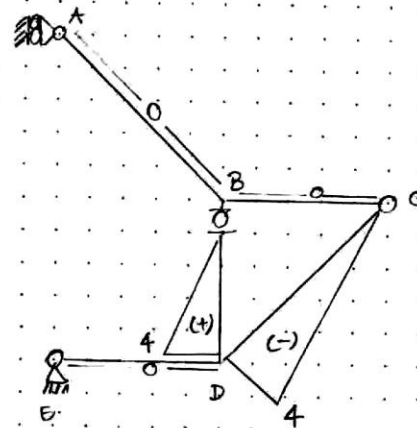
El corte de BC es la fuerza axial en la barra ED:

$$N_{ED} = -\frac{1}{3}$$

- Sistema Dos



$\cdot \times 2$



PREPARADOR: REDEGAR J. V. RUIZ

$$\sum M_E^A = 0 \Rightarrow -8A_h = 0 \Rightarrow A_h = 0 \Rightarrow M_{BA} = 0$$

$$\uparrow \sum F_v = 0 \Rightarrow E_v = 0 \Rightarrow M_{DE} = 0$$

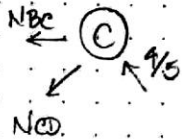
$$\sum M_D^B = 0 \Rightarrow 1 \times 4 - M_{DB} = 0 \Rightarrow M_{DB} = 4$$

Proyecto TEMA 3- MÉTODO DE LAS FUERZAS.

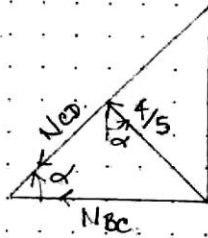
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NODO "C"



$\sin \alpha = 4/5$
 $\cos \alpha = 3/5$



$N_{cd} = +4/5 x - 4 \Rightarrow V_y(x) = 4/5$

$\uparrow \sum F_y = 0 \Rightarrow +4/5 x \cos \alpha - N_{cd} x \sin \alpha = 0 \Rightarrow N_{cd} = 3/5$

$\rightarrow \sum F_x = 0 \Rightarrow -N_{bc} - N_{cd} x \cos \alpha - 4/5 x \sin \alpha = 0$

$N_{bc} = -1$

$N_{bd} = 0$

$D_{10} = \frac{1 \times (-2400) \times 3}{3EI_0} + \frac{1 \times (400) \times 3}{6EI_0} + \frac{(-1) \times (-300) \times 5}{6EI_0} + \frac{10^{-5} \times 10^6}{EI_0} \times 5 \times \left[\frac{4}{5} \times 5 + (-\frac{1}{3}) \times 4 \right] +$
 $\frac{10^{-5} \times 10^6}{0,80 \times EI_0} \times \left[\frac{10 \times 1 \times 3}{2} + (-10) \times (-1) \times \frac{5}{2} \right]$

$D_{10} = -1450/EI_0$

$d_{11} = \frac{1 \times 1 \times 3}{3EI_0} + \frac{(-1) \times (-1) \times 5}{3EI_0} = \frac{8}{3EI_0}$

$d_{12} = \frac{(-1) \times (-1) \times 5}{6EI_0} = \frac{10}{3EI_0}$

$D_{20} = \frac{(-1) \times (-300) \times 5}{3EI_0} + \frac{10^{-5} \times 10^6}{EI_0} \times 5 \times \left[\frac{3}{5} \times 5 + (-1) \times 3 \right] + \frac{10^{-5} \times 10^6}{0,80 \times EI_0} \times \left[\frac{10 \times 4 \times 4}{2} + (-10) \times (-1) \times \frac{5}{2} \right]$

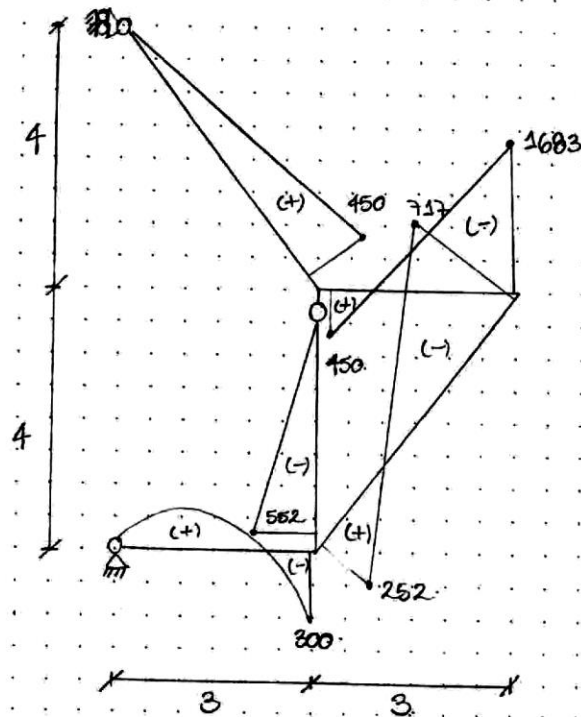
$D_{20} = 4250/EI_0$

$d_{22} = \frac{4 \times 4 \times 4}{3EI_0} + \frac{(-4) \times (-4) \times 5}{3EI_0} = \frac{48}{EI_0}$

$$\begin{cases} 0 = -1450/EI_0 + \frac{8}{3EI_0} \cdot X_1 + \frac{10}{3EI_0} \cdot X_2 & X_1 = 717 \text{ Kg-m} \\ 0 = 4250/EI_0 + \frac{10}{3EI_0} \cdot X_1 + \frac{48}{EI_0} \cdot X_2 & X_2 = -138 \text{ Kg} \end{cases}$$

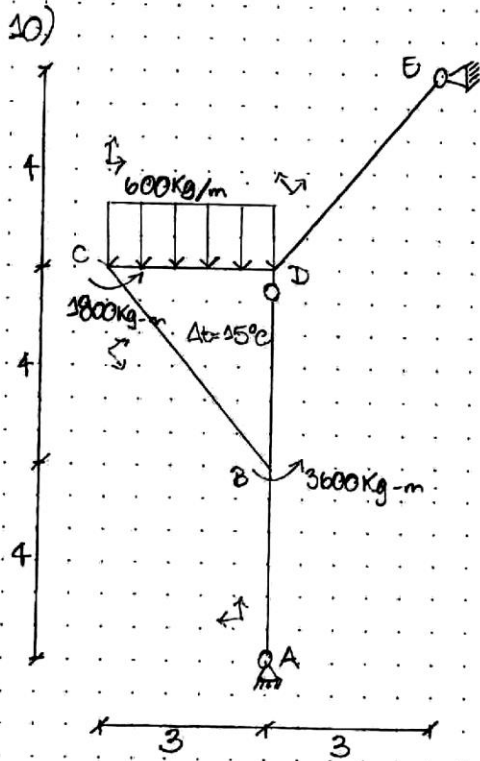
PREPARADOR: RODRIGUEZ J. VIREL R

DIAGRAMAS FINALES



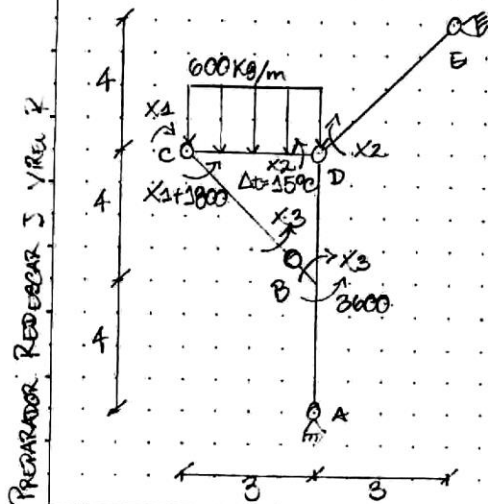
[Kg-m]

PREPARADOR: REZBERGAS J. VÍFEL R.



MÉTODO DE LAS FUERZAS.

- Por inspección se identifica un sistema hiperestático de tercer grado.
- Sistema Primario.

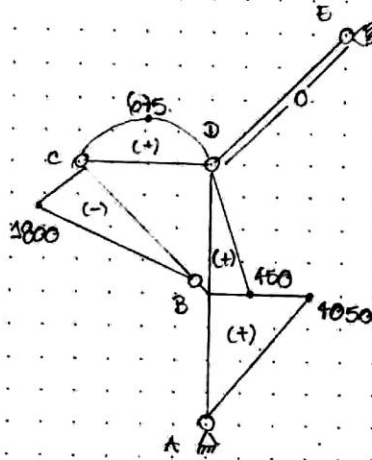
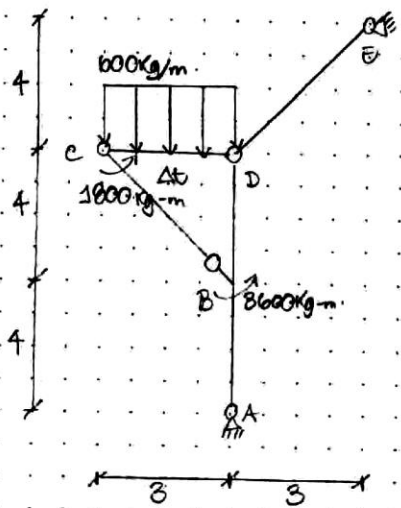


Ec. de Compatibilidad:

El trabajo realizado por los pares de fuerzas X_1 , X_2 y X_3 es cero.

$$D_1 = D_2 = D_3 = 0$$

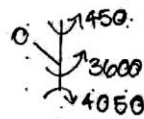
- Sistema Cero



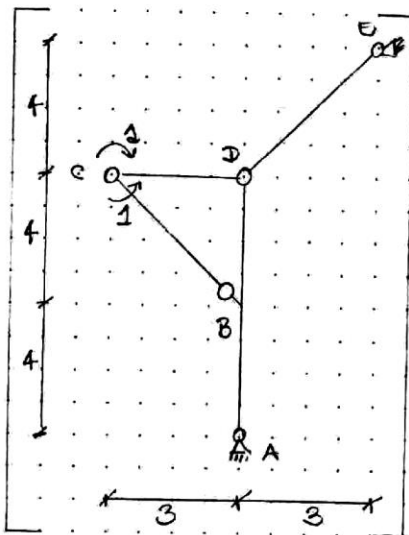
$$\sum M_{D \downarrow} = 0 \Rightarrow 600 \times 3 \times \frac{3}{2} + 1800 + 3600 + A_h \times 8 = 0 \Rightarrow A_h = -1012,5 \text{ Kg}$$

$$\sum M_{B \uparrow} = 0 \Rightarrow M_{BA} + A_h \times 4 = 0 \Rightarrow M_{BA} = 4050 \text{ Kg-m}$$

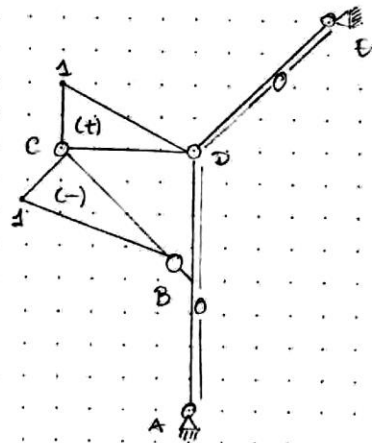
EQUILIBRIO DE JUNTA B.



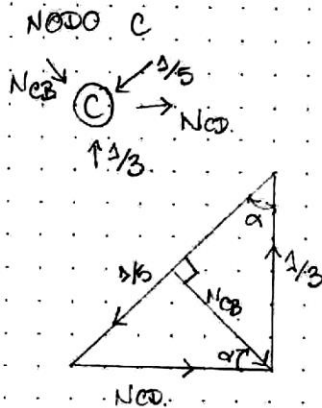
- Sistema Uno



• X4



PROFESOR REDEBENS I VIREL R



$$M3(x) = -\frac{1}{3}x + 1 \Rightarrow V3(x) = -\frac{1}{3}$$

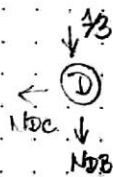
$$M3(x) = \frac{1}{6}x - 1 \Rightarrow V3(x) = \frac{1}{6}$$

$$\sin \alpha = \frac{1}{5}; \cos \alpha = \frac{4}{5}$$

$$\uparrow \sum F_v = 0 \Rightarrow \frac{1}{3} - \frac{1}{5} \times \cos \alpha - N_{CB} \times \sin \alpha = 0 \Rightarrow N_{CB} = \frac{4}{15}$$

$$\rightarrow \sum F_h = 0 \Rightarrow N_{CD} - \frac{1}{3} \times \sin \alpha + N_{CB} \times \cos \alpha = 0 \Rightarrow N_{CD} = 0$$

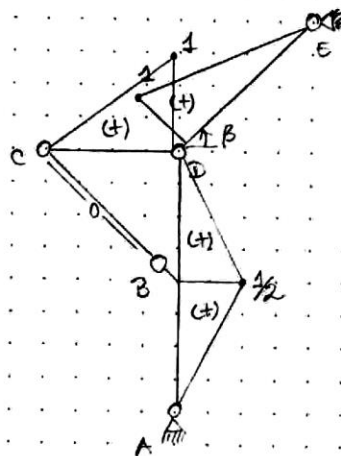
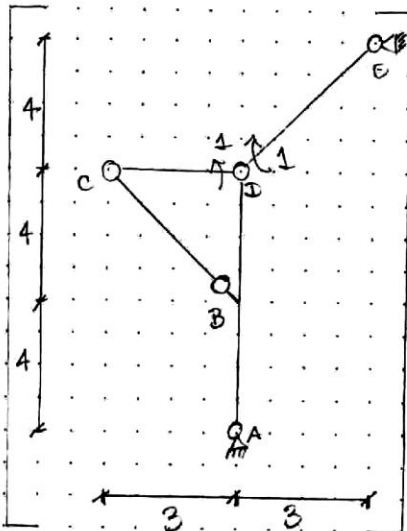
NODO D



Como E_h y $E_v = 0 \Rightarrow N_{DE} = 0$

$$\uparrow \sum F_v = 0 \Rightarrow N_{DB} = -\frac{1}{3}$$

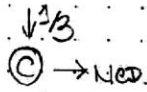
- Sistema DDB



$$\sum M_D = 0: 1 + A_h \times 8 = 0 \Rightarrow A_h = -\frac{1}{8}; \sum M_B = 0 \Rightarrow M_{BA} + A_h \times 4 = 0 \Rightarrow M_{BA} = \frac{1}{2}$$

PREPARADOR: ROBERTO J. VILAR R.

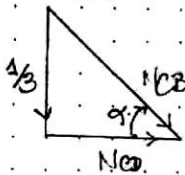
NODO "C"



$$M_3(x) = \frac{1}{3}x \Rightarrow V_3(x) = \frac{1}{3}$$

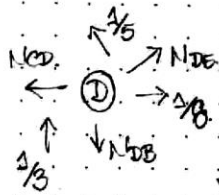
$$\uparrow \sum F_V = 0 \Rightarrow -\frac{1}{3} - N_{cB} \sin \alpha = 0 \Rightarrow N_{cB} = -\frac{5}{12}$$

$$\rightarrow \sum F_H = 0 \Rightarrow N_{cD} + N_{cB} \cos \alpha = 0 \Rightarrow N_{cD} = \frac{1}{4}$$



$$\sin \alpha = \frac{1}{5}; \cos \alpha = \frac{4}{5}$$

NODO "D"



$$M_3(x) = -\frac{1}{6}x + 1 \Rightarrow V_3(x) = -\frac{1}{6}$$

$$M_3(x) = -\frac{1}{6}x + \frac{1}{2} \Rightarrow V_3(x) = -\frac{1}{6}$$

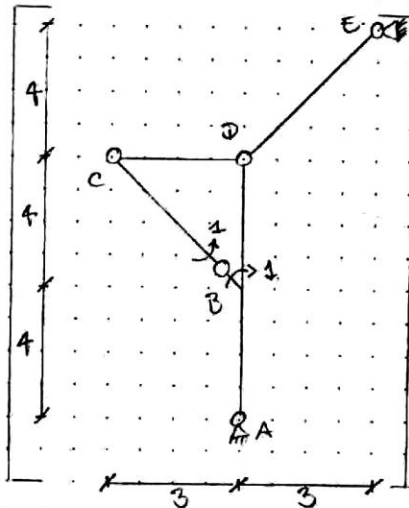
$$\rightarrow \sum F_H = 0 \Rightarrow -N_{cD} + \frac{1}{6} - \frac{1}{6} \times \sin \beta + N_{DB} \times \cos \beta = 0$$

$$N_{DB} = \frac{19}{40}$$

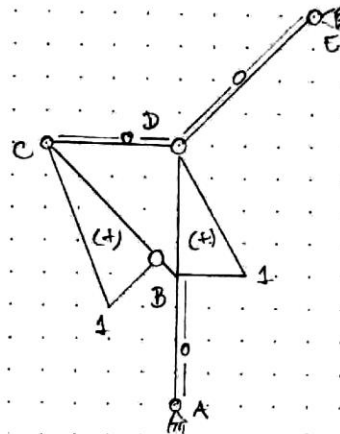
$$\uparrow \sum F_V = 0 \Rightarrow \frac{1}{3} + N_{DB} \times \sin \beta + \frac{1}{6} \times \cos \beta - N_{DB} = 0$$

$$N_{DB} = \frac{5}{6}$$

- Sistema Tres:



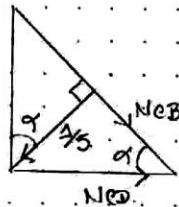
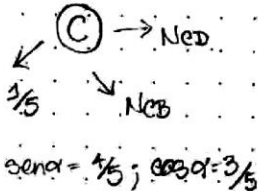
• x3.



TRENREIRO, PEDRO J. VIREL R.

NODO "C"

$$M_{3(x)} = \frac{1}{5}x \Rightarrow V_{3(x)} = \frac{1}{5}$$



$$\uparrow \sum F_V = 0 \Rightarrow -\frac{1}{5} \times \cos \alpha - N_{cB} \times \sin \alpha = 0$$

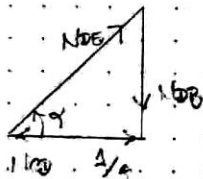
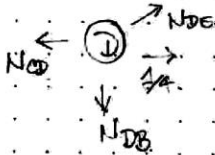
$$N_{cB} = -\frac{3}{20}$$

$$\rightarrow \sum F_H = 0 \Rightarrow N_{cD} + N_{cB} \times \cos \alpha - \frac{1}{5} \times \sin \alpha = 0$$

$$N_{cD} = -\frac{1}{4}$$

NODO "D"

$$M_{3(x)} = -\frac{1}{4}x + 1 \Rightarrow V_{3(x)} = -\frac{1}{4}$$



$$\rightarrow \sum F_H = 0 \Rightarrow -N_{cD} + \frac{1}{4} + N_{dB} \times \cos \alpha = 0$$

$$N_{dB} = 0$$

$$\uparrow \sum F_V = 0 \Rightarrow N_{DB} = 0$$

$$D_1 = D_{10} + d_{11} \cdot X_1 + d_{12} \cdot X_2 + d_{13} \cdot X_3$$

$$D_2 = D_{20} + d_{21} \cdot X_1 + d_{22} \cdot X_2 + d_{23} \cdot X_3 \quad \leftarrow \text{PRINCIPIO DE SUPERPOSICIÓN}$$

$$D_3 = D_{30} + d_{31} \cdot X_1 + d_{32} \cdot X_2 + d_{33} \cdot X_3$$

$$D_{10} = \frac{675 \times 1 \times 3}{6 \times 10^6} + \frac{(-1800) \times (-1) \times 5}{6 \times 10^6} + \frac{10^6 \times 10^{-5}}{E I_0} \times \frac{15}{2} \times \left[\frac{1}{15} \times 5 + \left(-\frac{1}{3} \right) \times 4 \right] + \frac{10^6 \times 10^{-5}}{E I_0} \times 0.70 \times \left[\frac{(-15) \times (-1) \times 5}{2} + 15 \times \frac{1 \times 3}{2} \right] \Rightarrow D_{10} = 4875 / E I_0$$

$$d_{11} = \frac{1 \times 1 \times 3}{6 \times 10^6} + \frac{(-1) \times (-1) \times 5}{6 \times 10^6} = \frac{8}{6 \times 10^6}$$

$$d_{12} = \frac{1 \times 1 \times 3}{6 \times 10^6} = \frac{1}{2 \times 10^6}$$

$$d_{13} = \frac{(-1) \times (-1) \times 5}{6 \times 10^6} = -\frac{5}{6 \times 10^6}$$

PREPARADOR PEDRO J. VIREZ Z.

$$D_{20} = \frac{675 \times 1 \times 3}{3EJ_0} + \frac{450 \times 0,5 \times 4}{3EJ_0} + \frac{4050 \times 0,5 \times 4}{3EJ_0} + \frac{10^{-5} \times 10^6}{EJ_0} \times \frac{15}{2} \times \left[\frac{-5}{12} \times 5 + \frac{1}{4} \times 3 + \frac{5}{6} \times 4 \right]$$

$$+ \frac{10^{-5} \times 10^6}{0,50 \cdot EJ_0} \times \left[\frac{15 \times 1 \times 3}{2} - \frac{15 \times \frac{1}{2} \times 4}{2} \right] \Rightarrow D_{20} = \frac{3975}{EJ_0}$$

$$D_{22} = \frac{1 \times 1 \times 3}{3EJ_0} + \frac{1 \times 1 \times 5}{3EJ_0} + \frac{1}{2} \times \frac{1}{2} \times \frac{4}{3EJ_0} \times 2$$

$$D_{22} = \frac{10}{3EJ_0}$$

$$D_{23} = \frac{1 \times \frac{1}{2} \times 4}{2 \cdot 3EJ_0} = \frac{2}{3EJ_0}$$

$$D_{30} = \frac{(-1800) \times (1) \times 5}{6EJ_0} + \frac{450 \times (1) \times 4}{3EJ_0} + \frac{10^{-5} \times 10^6}{EJ_0} \times \frac{15}{2} \times \left[\frac{-3}{20} \times 5 + \frac{1}{4} \times 3 \right]$$

$$+ \frac{10^{-5} \times 10^6}{0,50 \cdot EJ_0} \times \left[\frac{-15 \times 1 \times 5}{2} + \frac{(-15) \times \frac{1}{2} \times 4}{2} \right] \Rightarrow D_{30} = \frac{-2250}{EJ_0}$$

$$D_{32} = \frac{1 \times 1 \times 5}{3EJ_0} + \frac{1 \times 1 \times 4}{3EJ_0} = \frac{3}{EJ_0}$$

$$\left\{ \begin{array}{l} 0 = 4875 + \frac{8}{3} \cdot X_1 + \frac{1}{2} \cdot X_2 - \frac{5}{6} \cdot X_3 \\ 0 = 3975 + \frac{1}{2} \cdot X_1 + \frac{10}{3} \cdot X_2 + \frac{2}{3} \cdot X_3 \\ 0 = -2250 - \frac{5}{6} \cdot X_1 + \frac{2}{3} \cdot X_2 + 3 \cdot X_3 \end{array} \right\} \cdot \frac{1}{EJ_0}$$

$$X_1 = -1437 \text{ Kg-m}$$

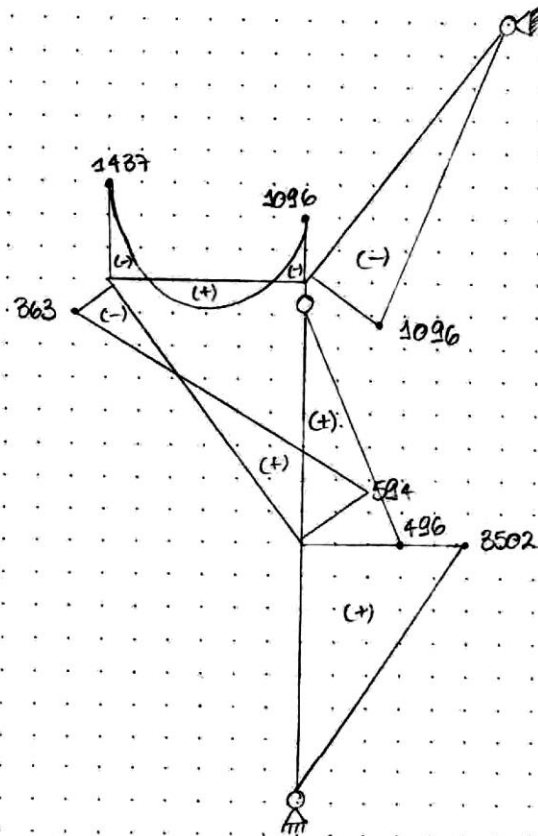
$$X_2 = -1096 \text{ Kg-m}$$

$$X_3 = 594 \text{ Kg-m}$$

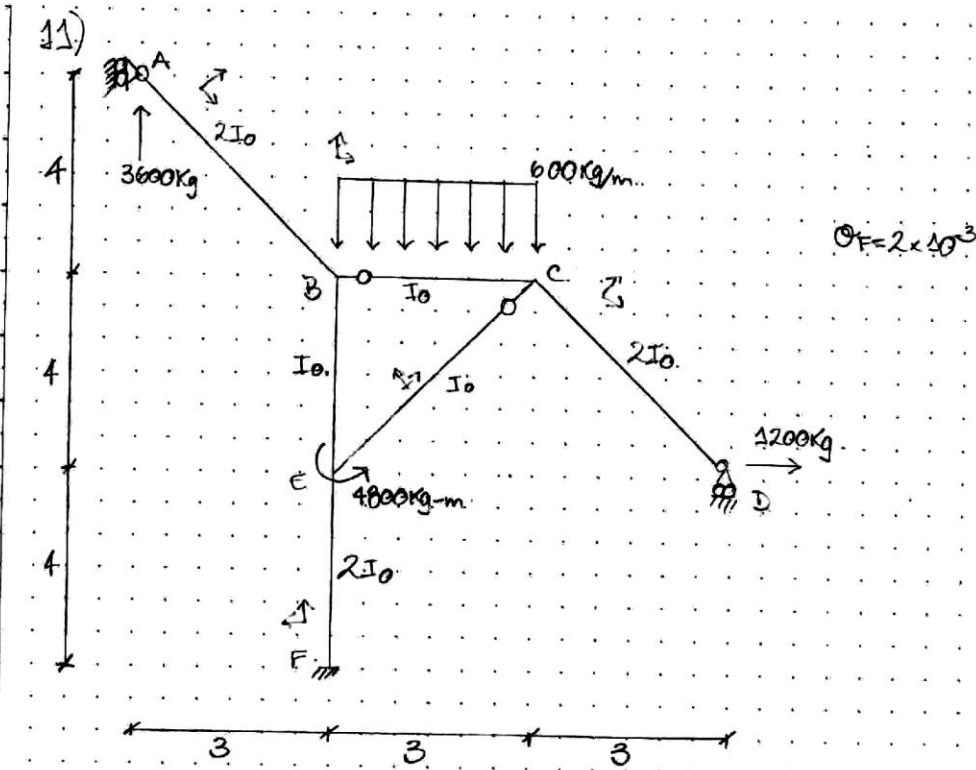
$$M_{BA} = M_{BA}^{(1)} + M_{BA}^{(2)} \cdot X_1 + M_{BA}^{(2)} \cdot X_2 + M_{BA}^{(3)} \cdot X_3 = 3502 \text{ Kg-m}$$

$$M_{BD} = M_{BD}^{(1)} + M_{BD}^{(2)} \cdot X_1 + M_{BD}^{(2)} \cdot X_2 + M_{BD}^{(3)} \cdot X_3 = 496$$

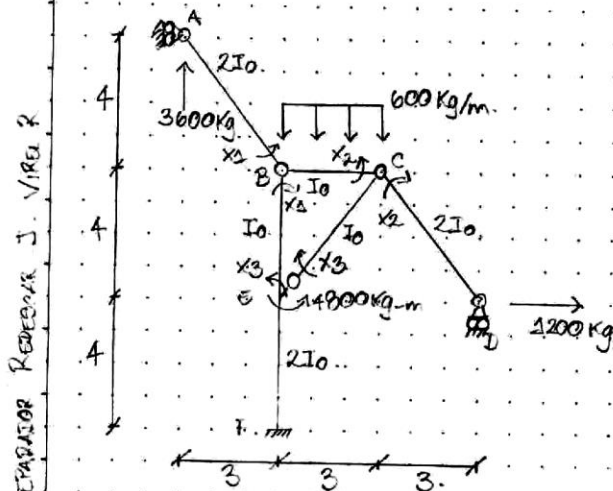
PREPARADOR: PEDROSAUR I. VIREL Z



PREPARADOR: REDEBOR J. VIREL R.



Por inspección se han identificado tres grados de hiperestaticidad.
MÉTODO DE LAS FUERZAS.
- Sistema Primario.



Ecuaciones de Compatibilidad

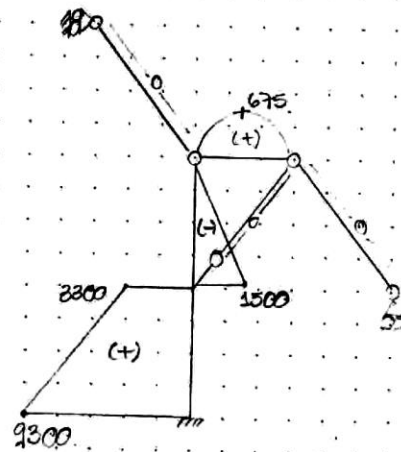
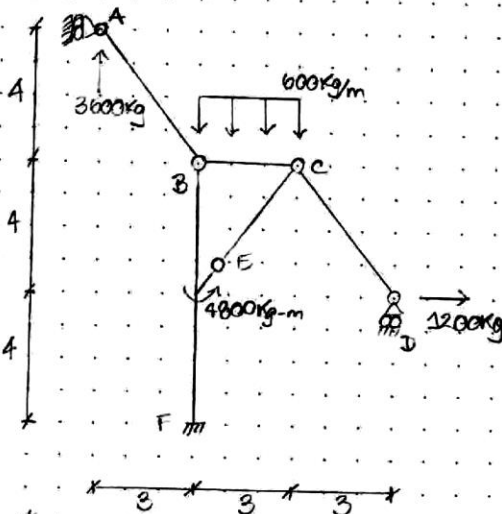
$$D_1 = D_2 = D_3 = 0$$

El trabajo realizado por las fuerzas

x_1 , x_2 y x_3 es cero.

PREPARADOR ROGERNAK J. VIREZ R

- Sistema Cero



$$\sum M_B = 0 \Rightarrow -3600 \times 3 - A_h \times 4 = 0 \Rightarrow A_h = -2700 \text{ Kg}$$

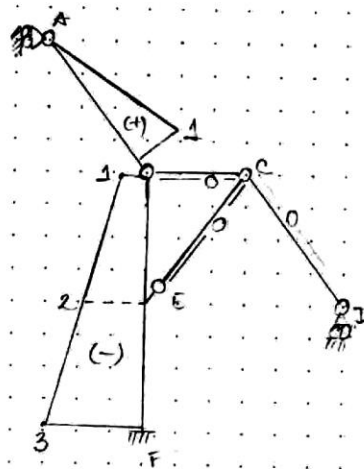
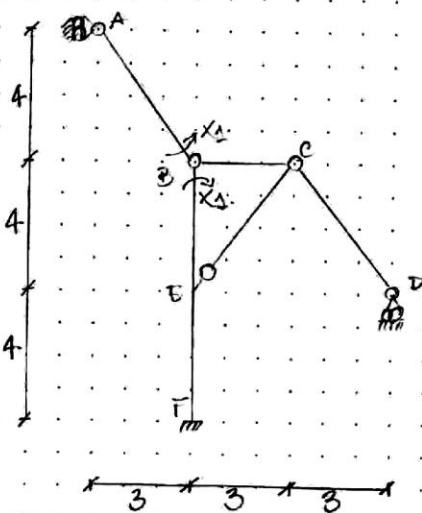
$$\sum M_C = 0 \Rightarrow 1200 \times 4 + D_v \times 3 = 0 \Rightarrow D_v = -1600 \text{ Kg}$$

$$\sum M_F = 0 \Rightarrow 4800 - 1200 \times 4 - 3600 \times 3 - 600 \times 3 \times \frac{3}{2} - M_F - A_h \times 12 + D_v \times 6 = 0$$

$$\sum F_h = 0 \Rightarrow A_h + 1200 + F_h = 0 \Rightarrow F_h = 1500 \text{ Kg} \quad M_F = 9300 \text{ Kg-m}$$

$$\sum M_E = 0 \Rightarrow M_{EF} - M_F + F_h \times 4 = 0 \Rightarrow M_{EF} = 3300 \text{ Kg-m}$$

- Sistema Uno



M. PARADOR, REVISOR, I. VIREL R.

Proyecto TEMA 3 - MÉTODO DE LAS FUERZAS

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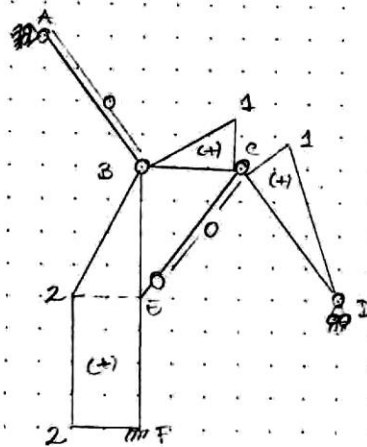
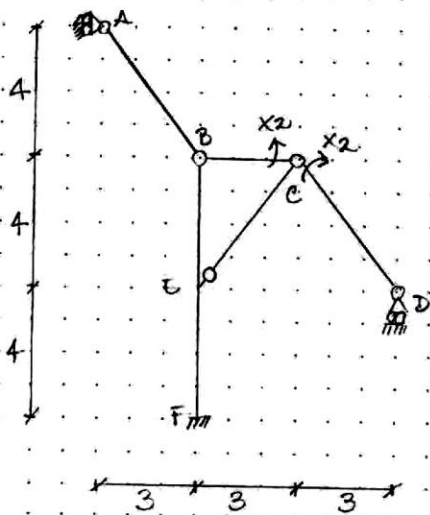
$$\sum M_B^{\text{ext}} = 0 \Rightarrow 1 - A_h \times 4 = 0 \Rightarrow A_h = 1/4 \text{ Kg.}$$

$$\sum M_F^{\text{ext}} = 0 \Rightarrow -M_F - A_h \times 4 \times 2 = 0 \Rightarrow M_F = -3 \text{ Kg-m}$$

$$\sum F_h^{\text{ext}} = 0 \Rightarrow A_h + F_h = 0 \Rightarrow F_h = -1/4 \text{ Kg}$$

$$\sum M_C^{\text{ext}} = 0 \Rightarrow M_{EF} - M_F + F_h \times 4 = 0 \Rightarrow M_{EF} = -2 \text{ Kg-m}$$

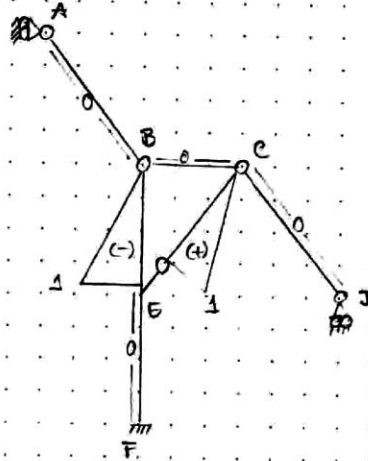
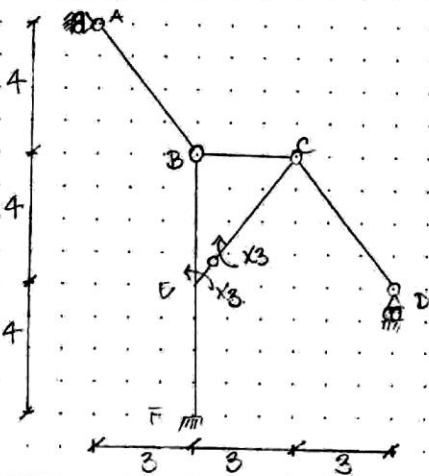
- Sistema Dos.



$$\sum M_C^{\text{int}} = 0 \Rightarrow -1 + D_v \times 3 = 0 \Rightarrow D_v = 1/3$$

$$\sum F_h^{\text{int}} = 0 \Rightarrow -M_F + D_v \times 6 = 0 \Rightarrow M_F = 2$$

- Sistema Tres.



PREPARADOR: REDEZOS J VIREL R

$$D_{10} + 3 \times 2 \times 10^{-3} \times \frac{10^6}{E I_0} = \frac{4}{6 \times 2 E I_0} \times [2 \times (-3) \times 9300 + 2 \times (-2) \times 3300 + 3300 \times (-3) + 9300 \times (-2)]$$

$$\frac{4}{6 E I_0} \times [2 \times (-2) \times (-1500) + (-1) \times (-1500)]$$

$$D_{10} = -33500 / E I_0$$

$$d_{11} = \frac{4}{6 \times 2 E I_0} \times [2 \times (-3) \times (-3) + 2 \times (-2) \times (-2) + 2 \times (-3) \times (-2)] + \frac{4}{6 E I_0} \times [2 \times (-2) \times (-2) + 2 \times (-1) \times (-1) + 2 \times (-1) \times (-2)]$$

$$+ \frac{1 \times 1 \times 5}{3 \times 2 E I_0} \Rightarrow d_{11} = 137 / 6 E I_0$$

$$d_{12} = \frac{4}{6 \times 2 E I_0} \times [2 \times (-3) \times (-2) + 2 \times (-2) \times (-2) + 2 \times (-3) \times (-2)] + \frac{4}{6 E I_0} \times [2 \times (-2) \times (-2) + 2 \times (-1) \times (-1)]$$

$$d_{12} = -50 / 3 E I_0$$

$$d_{13} = \frac{4}{6 E I_0} \times [2 \times (-2) \times (-1) + (-1) \times (-1)] \Rightarrow d_{13} = 10 / 3 E I_0$$

$$D_{20} - 2 \times 2 \times 10^{-3} \times \frac{30^6}{E I_0} = \frac{4}{6 \times 2 E I_0} \times [2 \times 9300 \times 2 + 2 \times 3300 \times 2 + 2 \times (3300 + 9300)] + \frac{(-1500) \times 2 \times 4}{3 E I_0}$$

$$D_{20} = 25200 / E I_0$$

$$d_{22} = \frac{2 \times 2 \times 4}{2 E I_0} + \frac{2 \times 2 \times 4}{3 E I_0} + \frac{1 \times 1 \times 3}{3 E I_0} + \frac{1 \times 1 \times 5}{3 \times 2 E I_0} \Rightarrow d_{22} = 91 / 6 E I_0$$

$$d_{23} = \frac{2 \times (-1) \times 4}{3 E I_0} \Rightarrow d_{23} = -8 / 3 E I_0$$

$$D_{30} = \frac{(-1500) \times (-1) \times 4}{3 E I_0} \Rightarrow D_{30} = 2000 / E I_0$$

$$d_{33} = \frac{(-1) \times (-1) \times 4}{3 E I_0} + \frac{(1) \times (1) \times 5}{3 E I_0} \Rightarrow d_{33} = 3 / E I_0$$

$$0 = -33500 + 137\frac{1}{6}x_1 - 50\frac{1}{3}x_2 + 10\frac{1}{3}x_3$$

$$0 = 25200 - 50\frac{1}{3}x_1 + 94\frac{1}{6}x_2 - 8\frac{1}{3}x_3$$

$$0 = 2000 + 10\frac{1}{3}x_1 - 8\frac{1}{3}x_2 + 3x_3$$

$\cdot \frac{1}{EIo}$

$$x_1 = 1534 \text{ Kg-m}$$

$$x_2 = -466 \text{ Kg-m}$$

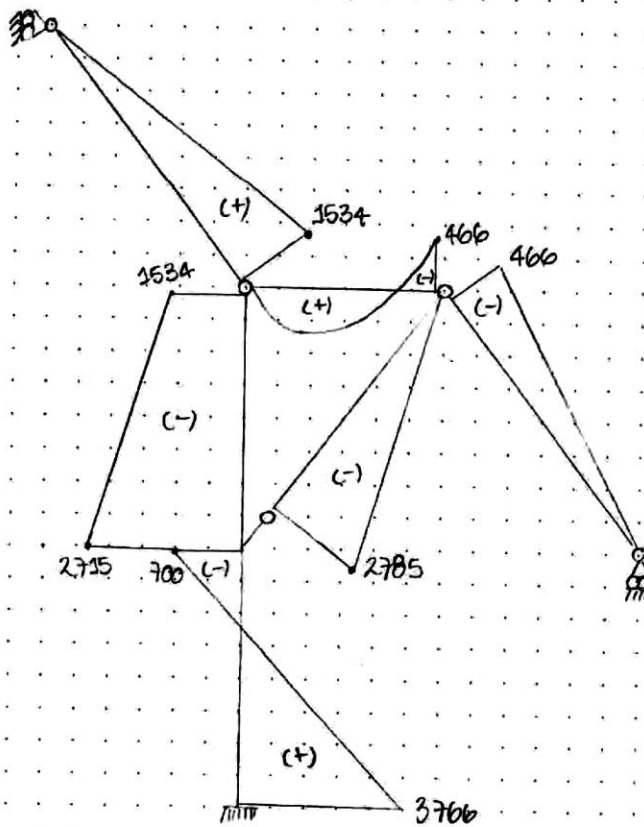
$$x_3 = -2785 \text{ Kg-m}$$

$$M_F = 9800 - 3x_1 + 2x_2 = 3766 \text{ Kg-m}$$

$$M_{EF} = 8800 - 2x_1 + 2x_2 = -700 \text{ Kg-m}$$

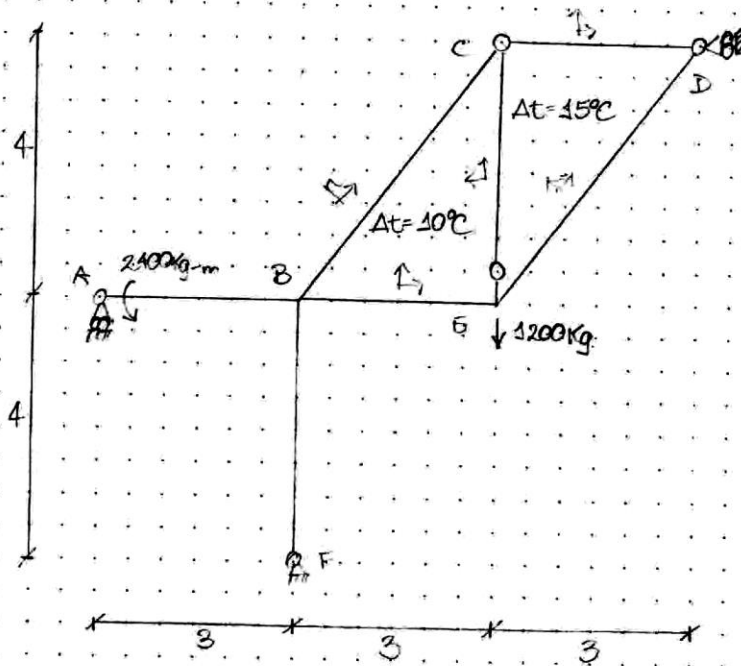
$$M_{EB} = -1500 - 2x_1 + 2x_2 - x_3 = -2715$$

DIAGRAMAS FINALES.



PREPARADOR: REDESAR J. VIREL R.

12) Grafique los Diagramas de Momento

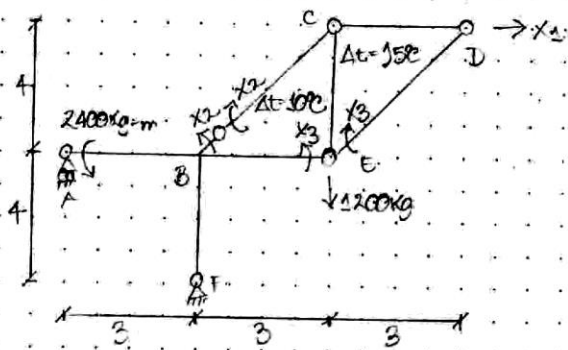


$I_D = 2 \times 10^{-2} m^4$

Todas las barras tienen el mismo Módulo de Rigidez a Flexión ($1000 kg \cdot m^2$)

MÉTODO DE LAS FUERZAS

Sistema Primario



Ecuaciones de Compatibilidad

El trabajo realizado por las fuerzas X_2 y X_3 es cero. En consecuencia:

$D_2 = D_3 = 0$

D_1 es el desplazamiento asociado al trabajo realizado por la fuerza X_1 :

$D_1 = 20.000 / E I_D$

PREPARADOR: REVISOR: J. VIREL R.

- Sistema Cero

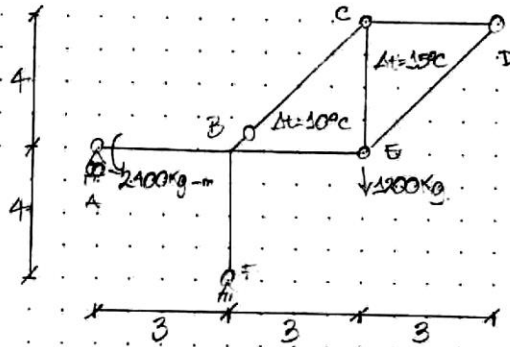
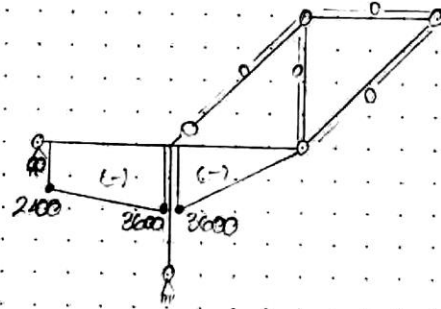


DIAGRAMA DE MOMENTOS



$$\sum \overset{+}{\curvearrowright} M_A = 0 \Rightarrow F_h = 0 \Rightarrow M_{BF} = 0$$

$$\sum \overset{+}{\curvearrowright} M_A = 0 \Rightarrow -A_v \times 3 + 2400 - 1200 \times 3 = 0$$

$$A_v = -400 \text{ kg}$$

$$\sum \overset{+}{\curvearrowright} M_B = 0 \Rightarrow M_{BA} + 2400 - A_v \times 3 = 0 \Rightarrow M_{BA} = -3600 \text{ kg-m}$$

- Sistema Uno

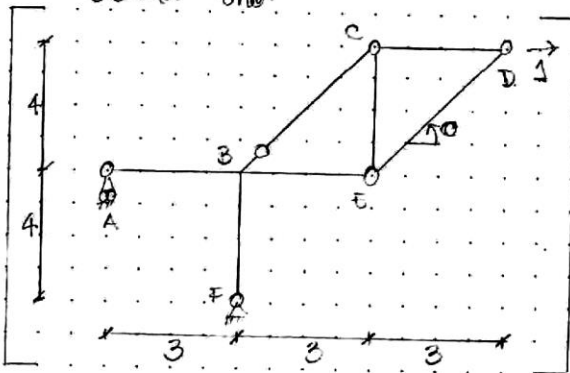
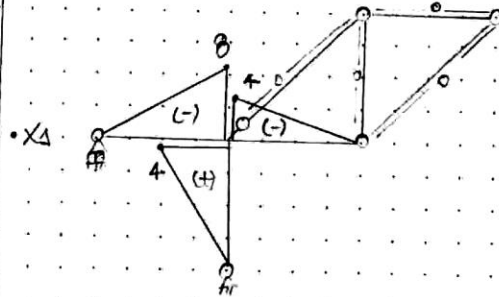


DIAGRAMA DE MOMENTOS



$$\sum \overset{+}{\curvearrowright} M_A = 0 \Rightarrow F_h + 1 = 0 \Rightarrow F_h = -1$$

$$\sum \overset{+}{\curvearrowright} M_A = 0 \Rightarrow -A_v \times 3 - 1 \times 3 = 0 \Rightarrow A_v = -8/3$$

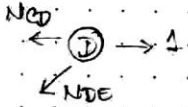
$$\sum \overset{+}{\curvearrowright} M_B = 0 \Rightarrow M_{BA} - A_v \times 3 = 0 \Rightarrow M_{BA} = -8$$

$$\sum \overset{+}{\curvearrowright} M_B = 0 \Rightarrow M_{BF} + F_h \times 4 = 0 \Rightarrow M_{BF} = 4$$

PREPARADOR: RODRIGUEZ, R.

NODO D

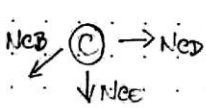
$$\sin \theta = 4/5 \quad ; \quad \cos \theta = 3/5$$



$$\sum F_x = 0 \Rightarrow -N_{DE} \sin \theta = 0 \Rightarrow N_{DE} = 0$$

$$\sum F_y = 0 \Rightarrow -N_{CD} - N_{DE} \cos \theta + 1 = 0 \Rightarrow N_{CD} = 1$$

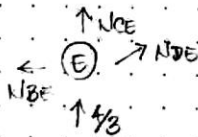
NODO C



$$\sum F_x = 0 \Rightarrow -N_{CB} \cos \theta + N_{CD} = 0 \Rightarrow N_{CB} = 5/3$$

$$\sum F_y = 0 \Rightarrow -N_{CB} \sin \theta - N_{CE} = 0 \Rightarrow N_{CE} = -4/3$$

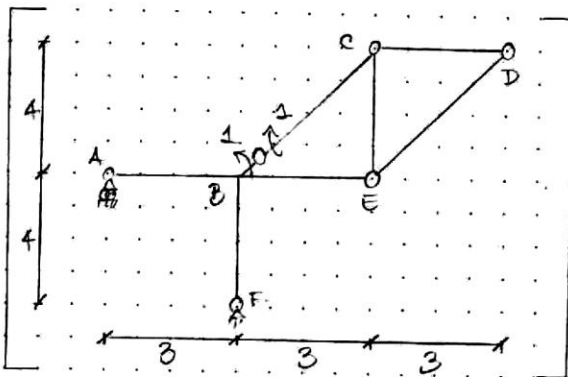
NODO E



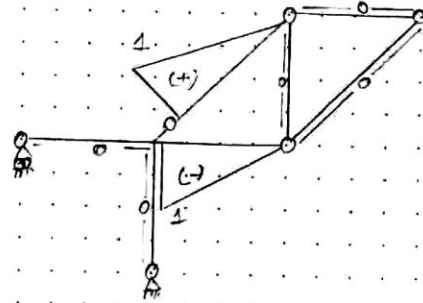
$$N_{3(x)} = 4/3 \times 4 = 4 \Rightarrow N_{2(y)} = 4/3$$

$$\sum F_x = 0 \Rightarrow -N_{BE} + N_{DE} \cos \theta = 0 \Rightarrow N_{BE} = 0$$

- Sistema JCB



$\times 2$



$$\sum F_x = 0 \Rightarrow F_H = 0 \Rightarrow M_{BE} = 0$$

$$\sum F_y = 0 \Rightarrow -A_v \times 3 + 1 - 1 = 0 \Rightarrow A_v = 0 \Rightarrow Y_{BA} = 0$$

PREPARADOR: REVISOR: J. V. S. R.

NODO D $\sin \theta = 4/5$; $\cos \theta = 3/5$

NODO D
 $\sum F_V = 0 \Rightarrow -N_{DE} \sin \theta = 0 \Rightarrow N_{DE} = 0$
 $\sum F_H = 0 \Rightarrow -N_{ED} - N_{DE} \cos \theta = 0 \Rightarrow N_{ED} = 0$

NODO C

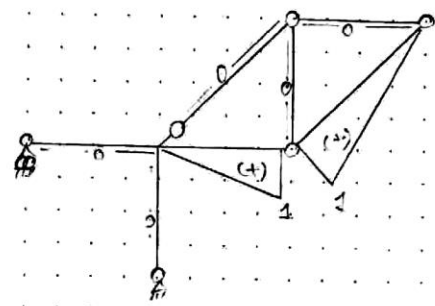
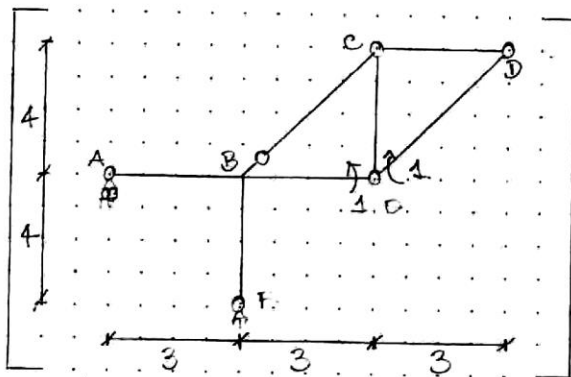
NODO C
 $M_{BC}(x) = -4/5x + 1 \Rightarrow V_{BC}(x) = -4/5$
 $\sum F_H = 0 \Rightarrow -N_{CB} \cos \theta + 1/5 \times 2 \sin \theta + N_{CD} = 0$
 $N_{CB} = 4/15$

$\sum F_V = 0 \Rightarrow -N_{CE} - N_{CB} \sin \theta - 1/5 \times \cos \theta = 0$
 $N_{CE} = -1/3$

NODO E

NODO E
 $M_{BE}(x) = 4/3x - 1 \Rightarrow V_{BE}(x) = 4/3$
 $\sum F_H = 0 \Rightarrow -N_{BE} + N_{DE} \cos \theta = 0$
 $N_{BE} = 0$

- Sistema Treg.



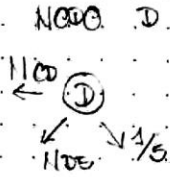
$\sum F_H = 0 \Rightarrow F_H = 0 \Rightarrow M_{BE} = 0$
 $\sum M_F = 0 \Rightarrow -A \times 3 + 1 - 1 = 0 \Rightarrow A_V = 0 \Rightarrow M_{BA} = 0$

PREPARADOR PEDERAP - 1º IPEL R.

Proyecto TEMA 3 - MÉTODO DE LAS FUERZAS

Fecha OCTUBRE 2012, Página 06

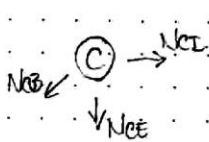
$\sin \alpha = 4/5$; $\cos \alpha = 3/5$
 $M_{3(x)} = -\frac{1}{5}x + 1 \Rightarrow V_{y(x)} = -\frac{1}{5}$



$\uparrow \sum F_V = 0 \Rightarrow -N_{10E} \cos \alpha - \frac{1}{5} \times \cos \alpha = 0 \Rightarrow N_{10E} = -3/20$

$\rightarrow \sum F_H = 0 \Rightarrow -N_{10D} - N_{10E} \cos \alpha + \frac{1}{5} \times \sin \alpha = 0 \Rightarrow N_{10D} = 1/4$

NODO C

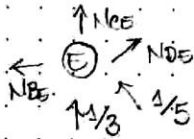


$\rightarrow \sum F_H = 0 \Rightarrow -N_{10C} \cos \alpha + N_{10S} = 0 \Rightarrow N_{10C} = 5/12$

$\uparrow \sum F_V = 0 \Rightarrow -N_{10E} \sin \alpha - N_{10C} = 0 \Rightarrow N_{10E} = -1/3$

NODO E

$M_{3(x)} = \frac{1}{5}x$ $V_{y(x)} = 1/3$



$\rightarrow \sum F_H = 0 \Rightarrow -N_{10E} + N_{10S} \cos \alpha - \frac{1}{5} \times \sin \alpha = 0$

$N_{10E} = -1/4$

$D_{10} = \frac{(-3600) \times (-4) \times 3}{3EJ_0} + \frac{3}{6EJ_0} \times (2 \times (-8)) \times (-3600) + (-8) \times (-2400)$

$+ \frac{10^5 \times 10^6}{EJ_0} \times \left[\frac{15}{2} \times 1 \times 3 + 5 \times \frac{5}{3} \times 5 + \frac{25}{2} \times (-\frac{1}{3}) \times 4 \right] + \frac{10^5}{0.80} \times \frac{10^6}{EJ_0} \left[\frac{(-10) \times (-4) \times 3}{2} \right]$

$D_{10} = 58975/EJ_0$

$d_{11} = \frac{(-8) \times (-8) \times 3}{3EJ_0} + \frac{(-4) \times (-4) \times 3}{3EJ_0} + \frac{4 \times 4 \times 4}{3EJ_0} = 304/3EJ_0$

$d_{12} = \frac{(-1) \times (-4) \times 3}{3EJ_0} \times \frac{4}{EJ_0}$

$d_{13} = \frac{(-4) \times (1) \times 3}{6EJ_0} = -\frac{2}{EJ_0}$

PREPARADOR PEDRO R. YIREL R.

Proyecto **TEMA 8 - MÉTODO DE LAS FUERZAS**

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$$D_{20} = \frac{(-3000) \times (-1) \times 3}{2EI_0} + \frac{10^{-5} \times 10^6}{EI_0} \times \left[5 \times \frac{4}{15} \times 5 + \frac{25}{2} \times \left(-\frac{1}{3}\right) \times 4 \right] + \frac{10^{-3} \times 10^6}{0,5EI_0} \times \left[\frac{10 \times 1 \times 5}{2} - \frac{10 \times (-1) \times 3}{2} \right]$$

$$d_{22} = \frac{1 \times 1 \times 5}{3EI_0} + \frac{(-1) \times (-1) \times 3}{2EI_0} = \frac{8}{3EI_0}$$

$$D_{20} = 4300/EI_0$$

$$d_{23} = \frac{(-1) \times 1 \times 3}{0,5EI_0} = -\frac{1}{2EI_0}$$

$$D_{30} = \frac{(-3600) \times 1 \times 3}{0,5EI_0} + \frac{10^{-5} \times 10^6}{EI_0} \times \left[\frac{15}{2} \times \left(-\frac{3}{20}\right) \times 5 + \frac{15}{2} \times \frac{1}{4} \times 3 + 5 \times \frac{5}{12} \times 5 + \frac{25}{2} \times \left(-\frac{1}{3}\right) \times 4 + 5 \times \left(-\frac{1}{4}\right) \times 3 \right]$$

$$+ \frac{10^{-2} \times 10^6}{0,5EI_0} \times \left[\frac{-10 \times (1) \times 3}{2} - \frac{15 \times 1 \times 5}{2} \right]$$

$$D_{30} = -2950/EI_0$$

$$d_{32} = \frac{1 \times 1 \times 3}{3EI_0} + \frac{1 \times 1 \times 5}{3EI_0} = \frac{8}{3EI_0}$$

$$\begin{cases} 20000 = 53975 + \frac{304}{3}x_1 + 4x_2 - 2x_3 \\ 0 = 4300 + 4x_1 + \frac{8}{3}x_2 - \frac{x_3}{2} \\ 0 = -2950 - 2x_1 - \frac{x_2}{2} + \frac{8}{3}x_3 \end{cases} \cdot \frac{1}{EI_0}$$

$$x_1 = -280 \text{ Kg}$$

$$x_2 = -1062 \text{ Kg-m}$$

$$x_3 = 697 \text{ Kg-m}$$

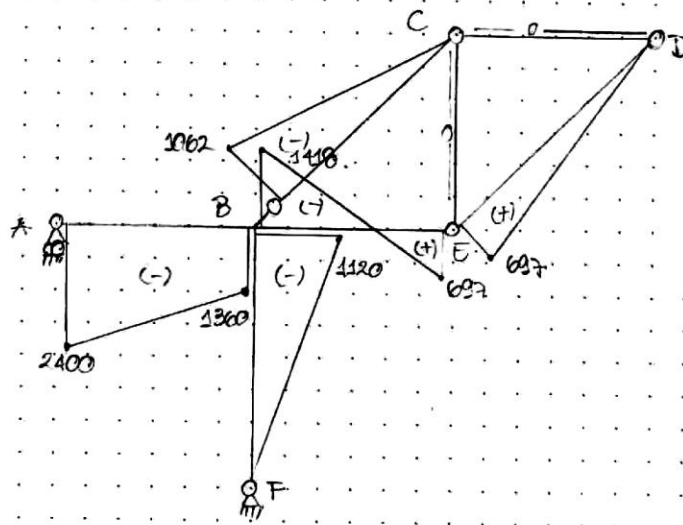
$$M_{BA} = -3600 - 8x_1 = -1360 \text{ Kg-m}$$

$$M_{BE} = -3600 - 4x_1 - x_2 = -1418 \text{ Kg-m}$$

$$M_{DE} = 4x_1 = -1120 \text{ Kg-m}$$

PREPARADOR: REDESOP 2. 1. 1. 2

DIAGRAMAS FINALES



PREPARADOR: P. JESSICA J. VIREL R.